Triangulating Multinationals and Trade

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Abstract

I develop a quantitative model of trade and MP that allows for a complete range of multinational activities, including selling only in the host country (horizontal FDI), selling only in the country of origin (vertical FDI), and selling to third-party countries (export platform FDI). Using the model and the available data, I bound outcomes, such as welfare, from various counterfactual scenarios without imposing specific assumptions about multinationals. As a theoretical matter, I prove that, for any country, gains from openness are at a maximum with horizontal FDI and at a minimum with vertical FDI. Empirical results show that a wide range of outcomes are consistent with the data. While the incompleteness of the data leaves open a wide range of outcomes, I show an example where the data available can still brings a clear judgment on a policy.

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1 Introduction

Modern foreign economic policies simultaneously involve both trade and multinational firms. For exmple, the implementation of Brexit resulted in higher tariffs and investment costs between the UK and the EU. Sanctions against Russia, which started in 2022, involve sanctions on exporters and multinational firms operating in Russia. Another policy, known as the Trans-Pacific Partnership, involves 11 countries across the Pacific, includes tariff reductions and a clause to facilitate foreign investment. Similar clauses can be found in the Transatlantic Trade and Investment Partnership negotiated by the EU and the US. What is the economic consequence of the aforementioned policies? Did these policies benefit the countries that implemented them? The scale and political importance of these multifaceted policies call for a quantitative framework to evaluate the counterfactual outcomes of these policies.

The complexity surrounding such a framework arises out of the potential interaction of trade and multinational production (MP), where we do not have a clear answer from the data. Namely, we do not know whether trade substitute MP, or trade complement MP. The available data only allow us to study trade and MP in isolation, and as a result we only know how firms trade and how firms produce abroad, but we do not know how firms invest and trade simultaneously.[1]

The lack of available data forces researchers to develop models with extreme assumptions on interactions. Some models assume horizontal foreign direct investment (FDI), which indicates that trade and MP are substitutes (e.g., Irarrazabal et al. (2013)). Some models assume vertical FDI, where the two are complements (e.g, Garetto (2013)). Some assume the export platform (e.g. Arkolakis et al. (2018)), which is more nuanced on the interaction but still prespecifies the interdependency.

These models provide dubious and overly precise policy predictions because the assumed interaction has direct implications on the consequence of policies. When the model assumes trade and MP are substitutes, the policy shock on trade will be mitigated by the expansion of multinational production. When the model assumes they are complements, the shock on trade will also impact MP. Assuming export platform FDI automatically implies that the policy shock on MP in a country

[1] A noticeable exception is the automobile industry. Head and Mayer (2019) use automobile data compiled by IHS Markit, whose sales data includes the country of headquarters, production, and sales for the majority of countries. As they acknowledge, data limitation remains a major challenge for the economy-wide model of multinationals.

will propagate to third countries where these foreign affiliates export.

Instead of providing a particular counterfactual outcome (e.g., welfare, trade) with significant assumptions on the link between trade and MP, I develop an alternative approach. I derive an interval of the outcomes with minimal assumptions on the interdependency of the two margins. Although this approach does not provide nonparametric prediction as in Adao et al. (2017), the approach allows flexible calibrations of the standard quantitative model of trade and MP. Each point of the interval encompasses a policy prediction from the calibration possible from the data. The interval summarizes the predictions from the model and the data. The noticeable feature of this approach is that the interval includes the outcome from past studies, and I can calculate the outcome as a point in the interval. Therefore, my approach can evaluate and compare the results from past studies in a unified manner. With this, I can highlight the direction of the potential bias stemming from these assumptions used in the past. The approach proceeds in three steps: (i) Restrict possible states of economy consistent with observed data, (ii) create a quantitative model for counterfactual experiments, and (iii) characterize a set of possible counterfactual outcomes that is consistent with the data.

I discuss *triangulation*, which is a core component of my empirical methodology, in Section 2. Triangulation connects the data and an allocation of MP, which is a crucial component of a quantitative model. The allocation, a concept I introduce in this study, is a three-dimensional array where each element is a production disaggregated by the firm origin, the production location, and the sales destination. Knowing the allocation is sufficient to calibrate the multinational Armington model – which I will develop in the later section – and perform counterfactual experiments using exact hat-algebra. For triangulation, I use widely available data on bilateral trade and MP. Although the data do not directly pin down the allocation, triangulation defines a set of allocations consistent with the observed data. Triangulation restricts the allocation by two accounting identities. The data on MP record the output in country l by firms from country i. If the allocation is summed over the destination, the summation must coincide with bilateral MP. The data on bilateral trade records the export from country l to country m. The summation must coincide with bilateral trade if the allocation is summed over the headquarters location. These accounting identities provide a set of allocations consistent with the data, which I define as *triangulated set*.

To fully utilize the power of triangulation, in Section 3, I develop a quantitative model compatible with triangulation. The model augments the Armington model of trade with MP. In the model, firms can produce their goods abroad and export them from their affiliates. The allocation and two elasticity parameters are sufficient to perform various counterfactual experiments. This model is compatible with the triangulation, as it accommodates any allocation in the triangulated set as an equilibrium outcome of the model. The model generalizes the models of quantitative export platform FDI such as Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018) and Wang (2021), and further incorporates horizontal FDI and vertical FDI as testable special cases. In addition, the model includes any hybrid of export platform FDI, horizontal FDI, and vertical FDI, demonstrating this modeling approach's greater flexibility.

In Section 4, I combine the model with triangulation to derive a *set* of possible counterfactual outcomes consistent with the data. The set comprises counterfactual outcomes from the model calibrated to some allocation in the triangulated set. One of the benefits of this approach is that the set is easy to characterize. If the outcome is scalar and is a continuous function of the allocation, the set of counterfactual outcomes forms an *interval.* The interval can be found by searching the allocation in the triangulated set that provides the counterfactual outcome's maximum and minimum.

This interval includes various models as points. Using this feature, I evaluate the quantitative implications of these models in the literature. I theoretically show that the intervals of gains from openness — the counterfactual welfare gain from having access to trade and MP — are characterized by the two canonical types of MP in the literature. One is pure horizontal FDI, where multinationals use affiliates to serve only the host country. The other is pure vertical FDI, where multinationals use affiliates to serve only the headquarters location. The allocation of pure horizontal FDI achieves the maximum of the gains from openness, and the allocation of pure vertical FDI achieves the minimum of the gains from openness. Gains from openness from any other allocations, including various export platform FDI, are between the values from the two types of MP.

Using the actual data, I show that the interval is economically large, and values from previous studies, which are the points in the interval, may be misleading. In Section 5, I perform counterfactual experiments for 15 countries using the World Input-Output Database and OECD Analytical AMNE Database. I provide an interval constructed from a triangulated set and values derived from the special cases for each country. The first three counterfactual outcomes are gains from openness, gains from trade, and gains from multinationals. All of these outcomes, especially gains from openness and gains from multinationals, indicate that the wide range of outcomes is consistent with the data; For example, in Germany, the gains from openness range from 4.5% to 14.5%, which is economically sizable. The wide range of the interval raises a concern about the robustness of previous studies, which assign a particular value within the interval based on the assumption of the allocation.

The broadness of the interval casts doubt on the usefulness of this approach. In Section 6, I exemplify how the interval is still helpful for policy evaluation even though the interval is so broad. Using the model from this study, I evaluate a hypothetical policy that penalizes offshoring by U.S. multinationals, which imitates an actual policy proposed by the President of the United States, Joe Biden. Through the lens of this model, the welfare consequence of the policy is conclusive for the U.S., showing that the policy does not improve the welfare of the U.S. people. I augment the data for U.S. foreign affiliates' sales to the U.S. from the Bureau of Economic Analysis and show that the U.S. may lose up to 0.2% in terms of welfare (the real wages). This approach helps with recommending a more desirable policy for the U.S. A broader policy penalizing U.S. MP, not only offshoring, potentially raises U.S. real wages by improving U.S. terms of trade. Concluding remarks are offered in Section 7, where I also discuss possible extensions of my approach.

The present study makes several contributions to studies on multinational firms. First, I show an alternative way to test horizontal FDI and vertical FDI[2]. Empirical studies are testing these two theories by performing regressions on FDI (Brainard, 1997 Markusen and Maskus, 2002, Carr et al., 2001, and Blonigen et al., 2003, Braconier et al. (2005) and Davies (2008)). My study provides an alternative testing strategy using a triangulated set.

Second, I construct a quantitative model that accommodates various types of MP: horizontal FDI, vertical FDI, export platform FDI and a mixture of the three. The models in the past studies prespecify the type of FDI. There are numerous studies on horizontal FDI —see, for example, Irarrazabal et al. (2013), Ramondo (2014), Gumpert et al. (2020), and McGrattan and Waddle (2020)—, and on vertical FDI, such as Garetto (2013) and Boehm et al. (2019). Ramondo and Rodríguez-Clare (2013), Tintelnot (2016), Arkolakis et al. (2018), Alviarez (2019), Head and Mayer

[2]Early theories of horizontal FDI and vertical FDI can be found in Markusen (1984) and Helpman (1984), respectively, and surveyed in Antràs and Yeaple (2014), Helpman (2006), and Yeaple (2013).

(2019), Garetto et al. (2019), Fan (2020), Li (2021) and Wang (2021) formulate affiliates as export platform FDI.

Among these studies, Ramondo and Rodríguez-Clare (2013) and Wang (2021) are the most relevant. My model extends a model developed in Ramondo and Rodríguez-Clare (2013). While the Ramondo and Rodríguez-Clare (2013) paper focuses on the export platform – where multinationals produce abroad to sell the goods to third countries – my model incorporates horizontal FDI and vertical FDI, which are distinct concepts discussed separately in the literature. I also point out that my model nests their model. Hence their counterfactual outcome will be a point in my prediction interval. Wang (2021) further extends Arkolakis et al. (2018) by incorporating a "bound approach" – an approach to use limited data and provide a bound (not a point) on counterfactual outcomes – developed by de Gortari (2020).[3] Wang (2021). He extends the model of Ramondo and Rodríguez-Clare (2013) and performs counterfactual experiments providing both the point (with some additional assumptions) and the bound. My study is independently developed and has several differences compared with Wang (2021). The set constructed in his study still precludes pure horizontal FDI and pure vertical FDI, which are crucial components in both the literature and my model. By including pure horizontal and vertical FDI in the model, I can theoretically characterize the maximum and the minimum of gains from openness, while Wang (2021) had to derive the bound numerically.

2 Triangulation

In this section, I introduce the idea of triangulation, which ties the data we observe to the state of the economy. I consider an economy with N countries indexed by $i, l, m \in \{1, ..., N\}$ and define a variable X_{ilm} as a gross output by country i's firm, produced in country l and consumed by a consumer in country m. From now on, I use these subscripts i, l and m as a general notion for firm origin, production location, and final destinaton, respectively. T_{lm} denotes a gross trade flow from country l to country m, and M_{il} denotes an output in country l by firms from country i (e.g, output in China by Japanese firms is denoted $M_{JPN,CHN}$). I denote the vector $\{X_{ilm}\}_{i=1,...,N,l=1,...,N,m=1,...N}$, $\boldsymbol{X}$ and call this an *allocation*. In my setting, the allocation, $\boldsymbol{X}$, is

[3] de Gortari (2020) develops an approach to numerically bound a counterfactual outcome in the model of the global value chain.

not observed. I observe vectors $\boldsymbol{T} \equiv \{T_{lm}\}_{l=1,\dots,N,m=1,\dots,N}$ and $\boldsymbol{M} \equiv \{M_{il}\}_{i=1,\dots,N,l=1,\dots,N}$, which are the data on bilateral trade and MP. While the allocation is not directly observed, I can use the data on bilateral trade and MP to restrict the possible allocation; I refer to this action as *triangulation*. To triangulate the allocation with the observed data, I use the following accounting identities:

$$M_{il} = \sum_{m=1}^{N} X_{ilm}$$
$$T_{lm} = \sum_{i=1}^{N} X_{ilm},$$

The first identity indicates that the total output of goods in country l by firms from country i, will be sold in some countries. The second identity implies that all the goods delivered from country l to country m must be produced by firms from some countries. Additionally, elements of the allocation must be non-negative because elements are gross values. These restrictions do not pin down the allocation; multiple allocations satisfy these restrictions [4]. However, these restriction can be still used to define a set of allocations consistent with the data. A triangulated set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is a set of allocations that satisfy the accounting identities and non-negativity. Specifically, any allocation $\boldsymbol{X}$ in $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ satisfies:

$$M_{il} = \sum_{m=1}^{N} X_{ilm}$$
$$T_{lm} = \sum_{i=1}^{N} X_{ilm}$$
$$0 \leq X_{ilm}$$

These equations restrict the set of allocations and hence partially identify the state of the economy.

3 Model

In this section, to make use of triangulation, I construct a model incorporating MP into the Armington model (Armington, 1969). In my model, firms can produce goods in foreign countries and

[4] There are N^3 elements in the allocation while there are only $2N^2$ constraints.

further export them. This model is isomorphic (provides the same aggregate implication) to the multinational Eaton-Kortum model as in Ramondo and Rodríguez-Clare (2013)[5].

There are N countries in the economy with representative firms and consumers. A consumer earns wages from her labor and purchase goods. A consumer in country l provides L_i amount of labor inelastically.

Goods are differentiated across the origin of the firm and the country of production. I denote C_{ilm} as the consumption of goods produced by a firm from country i, produced in country l and consumed in country m; I denote p_{ilm} as the price of such goods. The utility function of a representative consumer in country m is:

$$U_m = \left(\sum_{i=1}^{N} \left(\sum_{l=1}^{N} C_{ilm}^{\frac{\epsilon}{\epsilon+1}} \right)^{\frac{\epsilon+1}{\epsilon} \frac{\theta}{\theta+1}} \right)^{\frac{\theta+1}{\theta}}.$$

I assume $\theta < \varepsilon$, which implies that if the firm origin is the same, the goods are less differentiated than when compared to the goods produced by different firm origins. I define $\rho \equiv \frac{\epsilon-\theta}{\epsilon}$, where $0 \leq \rho < 1$ when $\theta < \epsilon$. The parameter θ is a usual trade elasticity, and the parameter ρ is a relative elasticity of substitution between the goods from the same firm origin and the goods from different firm origins. If $\rho = 0$, the goods from the same firm origin (but from different production locations) are as differentiated as goods from different firm origins. When $\rho \approx 1$, the goods from the same firm origin are close to perfect substitutes. The parameter ρ can be considered as a cannibalization parameter as in Ramondo and Rodríguez-Clare (2013).

The expenditure of goods, X_{ilm}, is:

$$X_{ilm} = \frac{P_{im}^{-\theta}}{\sum_{j=1}^{N} P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{ikm}^{-\theta/(1-\rho)}} X_m$$

where X_m denotes total absorption in country m and $P_{im} \equiv \left(\sum_{k=1}^{N} (p_{ikm}^{-\theta/(1-\rho)}) \right)^{-(1-\rho)/\theta}$ represents the price index in country m for goods produced by a firm from country i. The price index of

[5] For the time being, this paper abstracts intra-firm trade in Ramondo and Rodríguez-Clare (2013) because this model does not incorporate intermediate goods. The triangulation and the model with intermediate goods are shown in the appendix.

country m is:

$$P_m = \left[\sum_{i=1}^{N} P_{im}^{-\theta} \right]^{-1/\theta}.$$

I assume perfect competition; the price of the goods is the marginal cost of the goods. Labor is the only factor of production. Multinationals employ labor in country l to produce in country l. I denote the wage in country l w_l. A firm from country i requires an amount τ_{ilm} of labor to produce a unit of goods in country l and deliver to country m. The price of the goods p_{ilm} is:

$$p_{ilm} = w_l \tau_{ilm}.$$

Here $\tau_{ilm} \in (0, \infty)$ is a composite of productivity and various frictions (e.g., trade costs, knowledge transfer costs, and marketing costs) associated with MP and trade. When $\tau_{ilm} = \infty$, there is no possible technology to produce the goods for this purpose, hence $X_{ilm} = 0$.

In the model, there is a trade deficit, which is an exogenous transfer between countries. I denote D_m as the trade deficit of country m. The absorption of country m is a summation of labor income (total production) and trade deficit:

$$X_m = w_m L_m + D_m.$$

The market clearing condition is:

$$X_l = \sum_{k=1}^{N} \sum_{m=1}^{N} X_{klm} + D_l.$$

Denote the vectors of labor endowment, wage, labor requirement, τ, price and trade deficit as $\boldsymbol{L}$, $\boldsymbol{w}$,$\boldsymbol{\tau}$, $\boldsymbol{p}$ and $\boldsymbol{D}$. Given $\boldsymbol{L}, \boldsymbol{\tau}$ and $\boldsymbol{D}$, there is an equilibrium $\boldsymbol{w}$, $\boldsymbol{p}$, and $\boldsymbol{X}$ that satisfy the consumer optimization, the producer optimization and the market clearing condition.

3.1 Implications of the model

The novel feature of this model is that it rationalizes any allocation as an equilibrium outcome.

Proposition 3.1. *For any θ, ρ, and $\boldsymbol{X}$, there is a set of variables $(\boldsymbol{L}, \boldsymbol{D}, \boldsymbol{\tau})$ such that $\boldsymbol{X}$ is an* *outcome of the equilibrium of the model. If $\boldsymbol{L}$ and $\boldsymbol{D}$ are observed in addition, for any θ, ρ $\boldsymbol{L}$, $\boldsymbol{D}$, and $\boldsymbol{X}$, there exists $\boldsymbol{\tau}$ such that $\boldsymbol{X}$ is an outcome of the equilibrium of the model.*

Proof. See Appendix A. □

This feature of the model is neccesary for combining the model with triangulation. Triangulation restricts the allocation only from the data, and the model will not further restrict the possible allocation. In previous models in the literature, some allocations are precluded. For example, previous studies impose a structure on $\boldsymbol{\tau}$. Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018), and Fan (2020) assume $\boldsymbol{\tau}$ is the product of two bilateral costs: the cost of delivering knowledge from country i to country l, and the cost of delivering goods from country l to country m. This assumption implies $\tau_{ilm} = \gamma_{il}\xi_{lm}$, where γ_{il} denotes a knowledge cost, and ξ_{lm} denotes a trade cost. This specification identifies the allocation in the triangulated set, given a parameter ρ. For further usage, I denote the allocation from this specification $\boldsymbol{X}^{RRC}(\boldsymbol{T}, \boldsymbol{M}; \rho)$. This allocation results in proportionality; there exists a^1_{il} and a^2_{lm} such that $X_{ilm} = a^1_{il}a^2_{lm}$ for any i, l and m.

Wang (2021) assumes that $\boldsymbol{\tau}$ is a multiplication of three bilateral costs and is written as $\tau_{ilm} = \gamma_{il}\xi_{lm}\zeta_{im}$. In addition to the two costs specified in Ramondo and Rodríguez-Clare (2013), Wang adds ζ_{im}, a cost of marketing goods from country i to country m. Given a parameter ρ, this specification provides a set of allocations $\mathbb{X}^{Wang}(\boldsymbol{T}, \boldsymbol{M}; \rho)$. The allocation must be in the triangulated set and must also be rationalized as an equilibrium outcome of his model. Wang's set also results in proportionality. In Wang's model, there is a set of variables a^1_{il}, a^2_{lm}, and a^3_{im} such that $X_{ilm} = a^1_{il}a^2_{lm}a^3_{im}$ for any i, l and m.

Along with the explanation of proportionality restriction, I exemplify the restrictiveness of Wang's set. In Wang's model, the export of headquarters restricts the export of affiliates. Specifically, if the headquarters export to a country, their affiliates must also export to the country. [6] This restriction is why I refer to his model as an export platform model. In contrast, my model allows arbitrary patterns of sales for both headquarters and affiliates, such as pure horizontal FDI and pure vertical FDI.[7]

[6] Think of three countries: i, l and m. Suppose we consider an allocation with $X_{ilm} = 0$ and $X_{iim} > 0$. If the data indicates $M_{il} > 0$ and $T_{lm} > 0$, such allocation is not in the Wang's set. Specifically, if $X_{ilm} = 0$, γ_{il}, ξ_{lm} or ζ_{im} must be infinitely high. However, if $\zeta_{im} = \infty$, then it contradicts the fact that X_{iim} is positive. If $\gamma_{il} = \infty$, then it contradicts the data indicating M_{il} is positive. Similarly, if $\xi_{lm} = \infty$, then it contradicts the data indicating T_{lm} is positive.

[7] Appendix A shows that with a similar data structure, pure horizontal FDI and pure vertical FDI are excluded

This separability also excludes some plausible cost structures of MP. For example, people in the headquarters, the affiliate, and the sales destination may conduct a Zoom meeting. The Zoom meeting must be conducted when all the participants are awake. The possible time window depends on the trilateral time differences, rather than on two bilateral time differences; the cost of such a Zoom meeting is not multiplicatively separable.

3.2 Canonical theories of foreign direct investment

The model encompasses canonical theories of MP: pure horizontal FDI, pure vertical FDI and proportional export platform FDI. In pure horizontal FDI, all the output of the affiliates is sold to the host country (production location). In pure vertical FDI, all the output of the affiliates is sold to the firm origin (headquarters location).[8] In proportional export platform FDI, affiliates export to multiple countries, and the export of the affiliates (and headquarters) is proportional to the total export of the host country. Each theory pins down a specific allocation in the triangulated set.[9]

Denote the allocation of pure horizontal FDI $\boldsymbol{X}^{HFDI}(\boldsymbol{T},\boldsymbol{M})$. This allocation assumes all the output of affiliates is sold in the host country. Specifically, this allocation can be characterized by:

$$X_{ilm}^{HFDI} \equiv \begin{cases} M_{mm} - \sum_{k\neq m}^{N} T_{mk} & if\ i=m, l=m \\ T_{im} & if\ i=l, l\neq m \\ M_{im} & if\ i\neq l, l=m \\ 0 & otherwise. \end{cases}$$

Here, the output of affiliates from country i in country m is delivered to country m ($M_{im} = X_{imm}^{HFDI}$), and all the export from country i to country m is attributed to the export of firms from country i ($T_{im} = X_{iim}^{HFDI}$). The output of goods where the firm origin, production, and consumption are all in country m is the total output of headquarters in the country subtracting the export of the country ($X_{mmm}^{HFDI} = M_{mm} - \sum_{k\neq m}^{N} T_{mk}$). This allocation satisfies the accounting identities by construction. The allocation may contain a negative value. Therefore, it may not be in the triangulated set. For

from Wang's set.

[8] There are no intermediate goods in this setting. Vertical FDI indicates that the affiliates' production is sold to the headquarters location. The formulation with intermediate goods is discussed in the appendix.

[9] Given the proposition , there is always a $\boldsymbol{\tau}$ which rationalizes the allocation as an equilibrium outcome. Therefore, I do not discuss the assumption on $\boldsymbol{\tau}$, but I directly discuss the assumption on $\boldsymbol{X}$.

the allocation of pure horizontal FDI to be in the triangulated set, for all the countries, the output of the headquarters must be larger than the total export of the country:

$$\sum_{k\neq m}^{N} T_{mk} \leq M_{mm}$$

I denote the allocation of pure vertical FDI $\boldsymbol{X}^{VFDI}(\boldsymbol{T},\boldsymbol{M})$. This allocation assumes all the output of affiliates is sold to the headquarters' location. Specifically, this allocation can be characterized by:

$$X_{ilm}^{VFDI} = \begin{cases} T_{mm} & if\ i=m, l=m \\ T_{im}-M_{mi} & if\ i=l, i\neq m \\ M_{mi} & if\ i\neq l, i=m \\ 0 & if\ otherwise. \end{cases}$$

Here, the output of affiliates from country i in country m is all sold in country i ($M_{im} = X_{imi}^{VFDI}$). The export from country l to country m is a summation of the export of affiliates to the headquarters location and the export of headquarters ($T_{im} = X_{iim}^{VFDI} + X_{mim}^{VFDI}$). The output of goods where the firm origin, production, and consumption are all in country m is equal to the total output of goods produced and consumed in country m ($T_{mm} = X_{mmm}^{VFDI}$). This allocation satisfies the accounting identities by construction. The allocation is in the triangulated set if all the elements of the allocation are non-negative. For the allocation of the vertical FDI to be in the triangulated set, there must be more export from country i to country m than the output of firms from country m in country i:

$$M_{mi} \leq T_{im}$$

I denote the allocation of proportional export platform FDI $\boldsymbol{X}^{PFDI}(\boldsymbol{T},\boldsymbol{M})$. This allocation assumes that affiliates and headquarters have the same export intensity if they are located in the same country. Specifically, this allocation is described as:

$$X_{ilm}^{PFDI} = \frac{M_{il}}{\sum_{j=1}^{N} M_{jl}} T_{lm} \; \forall \; i, l, m.$$

This proportionality assumption is closely related to the separability assumption proposed by Ramondo and Rodríguez-Clare (2013). Specifically, when $\rho = 0$, these two assumptions provide the same allocation (See Appendix A). By definition, this allocation is always in the triangulated set.

3.3 Three measures of gains from multinationals and trade

To make use of the model, I introduce three values from the counterfactual experiments in the literature: gains from openness, gains from trade, and gains from multinationals.[10]

Gains from openness summarize the dependency of the country on trade and MP. Gains from openness of country q are changes in a real wage of country q, W_q, by moving to the current equilibrium from the counterfactual equilibrium without trade or MP. For the counterfactual experiments, I define a notion of real wage of country q, namely $W_q = \frac{w_q}{P_q}$ which is a utility measure of country q. Following Arkolakis et al. (2018), gains from openness for country q GO_q are expressed as: [11]

$$GO_q\left(\boldsymbol{X}\right) = \left(\frac{\sum_{l=1}^{N} X_{qlq}}{X_q}\right)^{-1/\theta} \left(\frac{X_{qqq}}{\sum_{l=1}^{N} X_{qlq}}\right)^{-(1-\rho)/\theta}.$$

Gains from openness can be decomposed into two parts. The first term captures the gains from foreign technology. With MP and trade, a consumer in country q can consumes goods produced by foreign firms, including the goods produced in country q. The second term captures gains from offshoring. With MP and trade, firms from country q can use foreign labor to offshore production and serve the consumer at home.

The gains from trade of country q are changes in the real wage of country q, W_q, by moving to the current equilibrium from the counterfactual equilibrium without trade (but with MP). I denote

[10] Derivation of gains from openness follow Arkolakis et al. (2018). Derivation of gains from trade and gains from multinationals are shown in Appendix A.

[11] Arkolakis et al. (2018) have profit margins. The implication of gains from openness does not change moving from perfect competition to monopolistic competition. Specifically gains from openness with monopolistic competitions is $GO_q\left(\boldsymbol{X}\right) = \left(\frac{\sum_{l=1}^{N} X_{qkq}}{X_q}\right)^{-1/\theta} \left(\frac{X_{qqq}}{\sum_{k=1}^{N} X_{qkq}}\right)^{-(1-\rho)/\theta} \frac{\sum_{l=1}^{N} M_{ql}}{M_{qq}}$. The third term only depends on the data, which is because the profit of firms are proportional to their sales.

it as GT_q:

$$GT_q(\boldsymbol{X}) = \left(\frac{\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilm} \right)^{\rho} X_{imm}^{(1-\rho)}}{X_m} \right)^{-1/\theta}.$$

Gains from multinationals of country q are a change in the real wage q, W_q, by moving to the current equilibrium from the counterfactual equilibrium without MP (but with trade). I denote it as GM_q:

$$GM_q(\boldsymbol{X}) = \left(\frac{\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilm} \right)^{\rho} X_{iim}^{(1-\rho)}}{X_m} \right)^{-1/\theta}.$$

For gains from multinationals, the counterfactual equilibrium differs from that of gains from openness and gains from trade. In the counterfactual equilibrium without MP, the wage is fixed to the level of the current equilibrium.[12] This assumption is required for the allocation to be sufficient to calculate gains from multinationals.[13]

4 Characterizing Counterfactuals

I focus on counterfactual experiments where the outcome can be calculated with $\boldsymbol{X}$, θ, and ρ. I assume θ and ρ are fixed and known.[14] I denote a scalar outcome of a counterfactual experiment $F : \boldsymbol{X} \to \mathbb{R}$ (e.g., gains from openness for country q, GO_q). Assuming a specific allocation provides a unique counterfactual outcome $F(\boldsymbol{X})$. Instead of assuming a specific allocation, I consider a set of allocations in the triangulated set and construct a set of possible outcomes:

$$\mathbb{F}(\boldsymbol{T}, \boldsymbol{M}) \equiv \{F(\boldsymbol{X}) \mid \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})\}.$$

I show that if the counterfactual function F is a continuous function of $\boldsymbol{X}$, the set can be characterized conveniently.

[12] Fixed wages can be rationalized by assuming some numeraire sector with free trade.

[13] Trade remains in the counterfactual equilibrium for gains from multinationals. Therefore, there is still an exchange of labor through trade; hence the change in relative wage must be taken into account (which is not the case for the other two since there is no trade). Tracking the change in relative wage requires solving the counterfactual equilibrium (which is not possible with only the allocation).

[14] The consequence of not knowing θ and ρ is discussed later.

Proposition 4.1. *If F is continuous on $\boldsymbol{X}(\boldsymbol{T},\boldsymbol{M})$ and $\mathbb{X}(\boldsymbol{T},\boldsymbol{M})$, then $\mathbb{F}(\boldsymbol{T},\boldsymbol{M})$ is an interval.*

Proof. The triangulated set $\mathbb{X}(\boldsymbol{T},\boldsymbol{M})$ is a set defined by a system of linear equations (without strict inequality). Therefore, the set is a connected closed convex set. Because the set is connected and F is a continuous function, $\mathbb{F}(\boldsymbol{T},\boldsymbol{M})$ must be connected. The connected set on $\mathbb{R}$ is an interval. □

All the counterfactual function F in this study is a continuous function of $\boldsymbol{X}$. If the interval is bounded, this proposition implies that calculating the maximum and minimum of the set $\mathbb{F}$ is sufficient to characterize the set of counterfactual outcomes. For the time being, I assume that the interval is bounded.[15] I can calculate the maximum of $\mathbb{F}$ by solving the following problem:

$$\begin{aligned} F^U &= \textit{maximize } F(\boldsymbol{X}) \text{ over } \boldsymbol{X} \\ s.t.\ M_{il} &= \sum_{m=1}^{N} X_{ilm}\ \forall\ i,l \\ T_{lm} &= \sum_{i=1}^{N} X_{ilm}\ \forall\ l,m \\ X_{ilm} &\geq 0\ \forall\ i,l,m. \end{aligned}$$

The minimum of $\mathbb{F}$ can be similarly calculated.

4.1 Gains from openness and the canonical theories

I define $\mathbb{GO}_q(\boldsymbol{T},\boldsymbol{M})$ as a set of gains from openness for country q that is consistent with the data. Because gains from openness are a continuous function of the allocation $\boldsymbol{X}$, the set is an interval:

$$\mathbb{GO}_q(\boldsymbol{T},\boldsymbol{M}) = [GO_q^L(\boldsymbol{T},\boldsymbol{M}), GO_q^U(\boldsymbol{T},\boldsymbol{M})].$$

I show that the allocations of pure horizontal FDI and pure vertical FDI are the limiting cases.

Theorem 4.1. *For any country q, if the pure horizontal FDI allocation is in the triangulated set, then $GO_q^U(\boldsymbol{T},\boldsymbol{M}) = GO_q\left(\boldsymbol{X}^{HFDI}(\boldsymbol{T},\boldsymbol{M})\right)$. If the pure vertical FDI allocation is in the triangulated set, $GO_q^L(\boldsymbol{T},\boldsymbol{M}) = GO_q\left(\boldsymbol{X}^{VFDI}(\boldsymbol{T},\boldsymbol{M})\right)$.*

[15] Practically, the interval may not be bounded. Appendix B shows how we can address the unboundedness of the interval for some cases.

The intuition is explained as follows. Note that $GO_q(\boldsymbol{X})$ is a decreasing function of X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$. Higher X_{qqq} implies higher demand for the domestic goods that remain available in autarky and higher $\{X_{qlq}\}_{l=1,..N,l\neq q}$ implies higher demand for offshored goods, which are more substitutable than the goods made by foreign firms. In the proof, I show that these two allocations are the extremes in terms of these variables. In the pure horizontal FDI allocation, there is no offshoring (minimize $\{X_{qlq}\}_{l=1,..N,l\neq q}$). Moreover, everything that foreign firms produce in country q is consumed in country q (minimize X_{qqq}). In the pure vertical FDI allocation, everything that firms from country q produce abroad is sent back to country q (maximize $\{X_{qlq}\}_{l=1,..N,l\neq q}$) and everything that foreign firms produce in country q is exported back, hence not consumed in country q (maximize X_{qqq}). Because $GO_q(\boldsymbol{X})$ is a decreasing function of both elements, these allocations characterize the maximum and the minimum of the interval. The formal proof is stated below.

Proof. I first show bounds for X_{ilm}. Formally, the bounds are inequalities that any X_{ilm} must satisfy if the allocation is in the triangulated set. While these bounds only use a subset of constraints, they are useful to calculate bounds for the gains from openness:

$$
\begin{aligned}
0 &\leq X_{ilm} \\
X_{ilm} &\leq M_{il} \\
X_{ilm} &\leq T_{lm} \\
M_{il} - \sum_{n\neq m}^{N} T_{ln} &\leq X_{ilm}.
\end{aligned}
$$

I use the subset of these bounds to construct bounds for X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$:

$$
\begin{aligned}
0 \leq X_{qlq} &\leq M_{ql}. \\
M_{qq} - \sum_{n\neq q} T_{qn} \leq X_{qqq} &\leq T_{qq}.
\end{aligned}
$$

I first show the case of GO_q^U. Note that maximizing gains from openness corresponds to minimizing X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$. I take the lower bound for each variable, which are:

$$0 \leq X_{qlq}$$
$$M_{qq} - \sum_{n\neq q} T_{qn} \leq X_{qqq}.$$

Any allocation that X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$ achieve these lower bounds (if in the triangulated set) achieves maximum gains from openness. The pure horizontal FDI allocation achieves the lower bound for each variable. By the assumption, the allocation is in the triangulated set. Therefore, the allocation achieves the maximum gains from openness.

A similar argument can be made for GO_q^L. Note that minimizing the gains from openness corresponds to maximizing X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$. I take the upper bounds for each variable which are

$$X_{qlq} \leq M_{ql}$$
$$X_{qqq} \leq T_{qq}.$$

Any allocation that X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$ achieve these upper bounds if the allocation is in the triangulated set, achieves the minimum for the gains from openness. The pure vertical FDI allocation achieves the upper bound for each variable. By the assumption, the allocation is in the triangulated set. Therefore, the allocation achieves the minimum for gains from openness.

□

Hereafter, for notational simplicity, I omit $(\boldsymbol{T}, \boldsymbol{M})$ from the notion of each specific allocation. If both the allocations of horizontal FDI and the vertical FDI are in $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$, then $\mathbb{GO}_q = \left[GO_q\left(\boldsymbol{X}^{VFDI}\right), GO_q\left(\boldsymbol{X}^{HFDI}\right)\right]$.

There is a few noticable implications from this theorem. First, the result does not depend on the specific parameter value of θ and ρ. Second, these two allocations achieve the maximum or minimum of gains from openness for any country simultaneously. If one assumes the allocation of pure horizontal FDI or the allocation of pure vertical FDI, the model achieves the maximum or the minimum of the gains from openness for all the countries. Third, this result crucially depends on the assumption that the allocation is in the triangulated set. The specific method for obtaining the maximum and the minimum if these allocations are not in the triangulated set is explained in the next section.

5 Quantification

I quantify various counterfactual experiments discussed in the literature and demonstrate the usefulness of this approach. For the benchmark result, I set $\theta = 4.5$ and $\rho = 0.55$ following Arkolakis et al. (2018).[16] Computational details are described in Appendix C.

I use two sets of data for bilateral trade and MP. The data on bilateral trade is taken from the World Input-Output Database, which is collected by 12 research institutes headed by the University of Groningen, The Netherlands, and covers trade in both goods and services (Timmer et al., 2015). By aggregating the input-output table over the purchase dimension, I construct the bilateral trade flow. For the data on bilateral MP, I use the analytical AMNE (Activity of Multinational Enterprises) Database constructed by OECD.[17] I use the data from 2013. I aggregate the industry dimension, which includes primary, manufacturing and services, and aggregate the data to 15 countries that appears in both data sets. The countries are Australia, Brazil, Canada, China, France, Germany, Indonesia, Italy, Japan, Mexico, Russia, Spain, the United Kingdom, the United States, and the rest of the world.

I firstly verify if the allocations of horizontal FDI and vertical FDI are in the triangulated set. The allocation of vertical FDI is not in the triangulated set, while the allocation of horizontal FDI is. The data refute vertical FDI and does not refute horizontal FDI.

5.1 Gains from openness

I utilize Theorem 4.1 to calculate the interval of gains from openness. The maximum gains from openness are calculated as $GO_q(\boldsymbol{X}^{HFDI})$. For the minimum gains from openness, the theory does not apply because the vertical FDI allocation is not in the triangulated set. I modify the vertical FDI allocation and verify that the modified allocation achieves the minimum if it is in the

[16] Without additional data on $\boldsymbol{X}$ and cost shifter / demand shifter, it is not possible to identify the parameter θ and ρ. Because $\boldsymbol{X}$ can be rationalized as an equilibrium outcome for any parameter θ and ρ, the allocation $\boldsymbol{X}$, in isolation, cannot identify θ and ρ; hence the data $(\boldsymbol{M}, \boldsymbol{T})$ cannot identify the parameter. The possible consequence of varying these parameters are discussed in Appendix B.

[17] The final product of analytical AMNE calculates the trade of MP, but it relies on some assumptions. I use the intermediate calculation of an analytical AMNE database, which calculates the bilateral flow of MP.

triangulated set. Denote this modified allocation $\boldsymbol{X}^{MVFDI,q}$. For each country q, I guess a subset of the allocation $\{X_{qlq}^{MVFDI,q}\}_{l=1,\ldots,N}$:

$$X_{qlq}^{MVFDI,q} = min\left(M_{ql}, T_{lq}\right),$$

which is a simple upper bound for the relevant variables. If an allocation satisfies this restriction and is in the triangulated set, gains from openness from this allocation are the minimum of gains from openness for country q. The existence of such an allocation can be verified using linear programming techniques, and the minimum is:

$$GO_q^L\left(\boldsymbol{X}^{MVFDI,q}\right) = \left(\frac{X_{qqq}^{MVFDI,q}}{X_q}\right)^{-(1-\rho)/\theta} \left(\frac{\sum_{k=1}^{N} X_{qkq}^{MVFDI,q}}{X_q}\right)^{-\rho/\theta}.$$

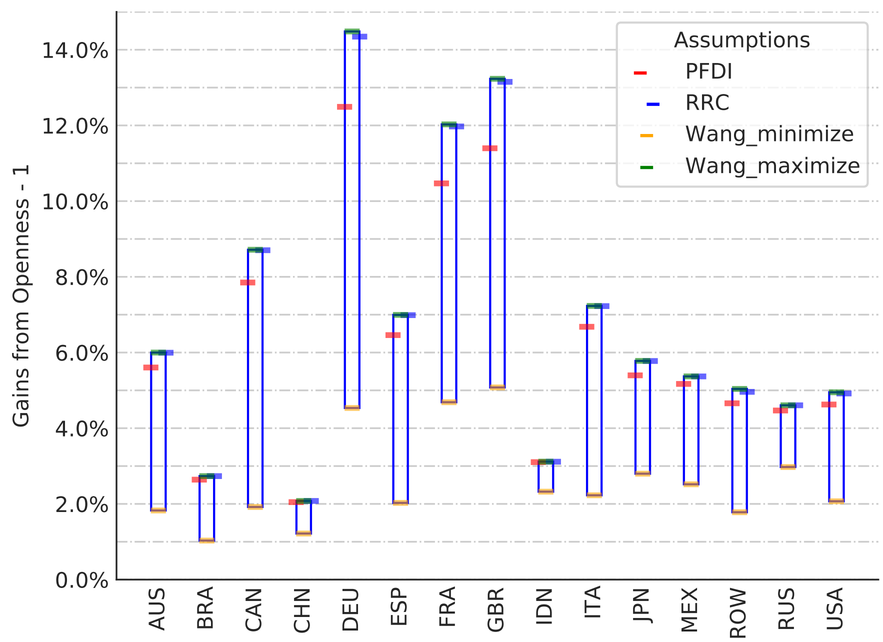
I verify that in the data there is an allocation in the triangulated set that achieves this value (for each country). I use the value as the minimum of gains from openness.[18]

In addition to the interval of gains from openness, I calculate values from four allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo Rodríguez-Clare's model, (iii) allocation from Wang's model that maximizes gains from openness (I denote this Wang-max allocation), and (iv), allocation from Wang's model that minimizes gains from openness (I denote this Wang-min allocation). Figure 1 shows the result. There is a wide range of gains from openness consistent with the data. For example, the gains from openness in Germany range from 4.5% to 14.5%. Generally, European countries, which are open both in trade and MP, tend to have wider intervals, compared with other countries. In most countries, the allocation from Ramondo and Rodríguez-Clare's model achieves a value close to the maximum. For example, the gains from openness of Germany with Ramondo and Rodríguez-Clare's model are 14.3%, which is almost the maximum of the interval. This tendency suggests that, in this data, the assumption by Ramondo and Rodríguez-Clare (2013) may overestimate gains from openness. The value from the proportional export platform FDI allocation is similar. The gains from openness of the U.S. with the proportional export platform FDI allocation are 4.6% while that of the U.S. with Ramondo

[18] There is a similar approach when horizontal FDI is refuted. Construct a subset of the modified horizontal FDI allocation, which is $X_{qlq}^{MHFDI,q} = max\left(0, M_{il} - \sum_{n\neq m} T_{ln}\right)$. If the allocation is feasible, such alocation achieves the maximum.

and Rodríguez-Clare's model are 4.9%. Wang's model seems to achieve the maximum and the minimum of the interval. The minimum and the maximum in Wang's model for Germany are 4.5% and 14.5%, respectively, which are equal to the minimum and the maximum achieved from the triangulation. Because gains from openness only depend on a small subset of the variables in the allocation, it is not surprising that Wang's model is sufficiently flexible to achieve the minimum and the maximum.

Figure 1: Gains from openness

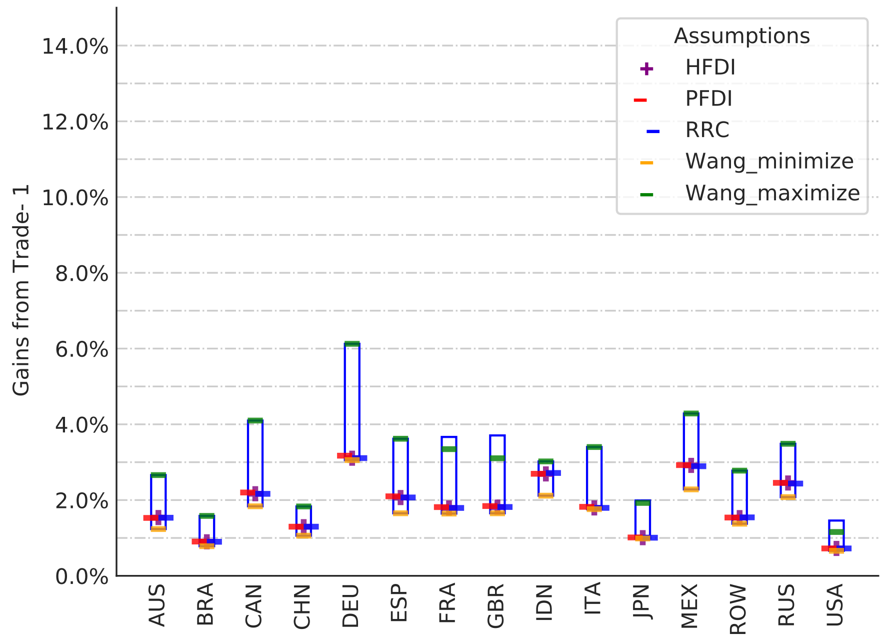


5.2 Gains from trade

Regarding gains from trade, I show the interval and the five values from the following allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo and Rodríguez-Clare's model, (iii) horizontal FDI allocation, (iv) allocation from Wang's model that maximizes gains from trade and (v) allocation from Wang's model that minimizes gains from trade. Figure 2 shows the result.

The range of the interval for gains from trade is significantly narrower than that of gains from openness. For example, while gains from openness in Germany range from 4.5% to 14.5%,

Figure 2: Gains from trade



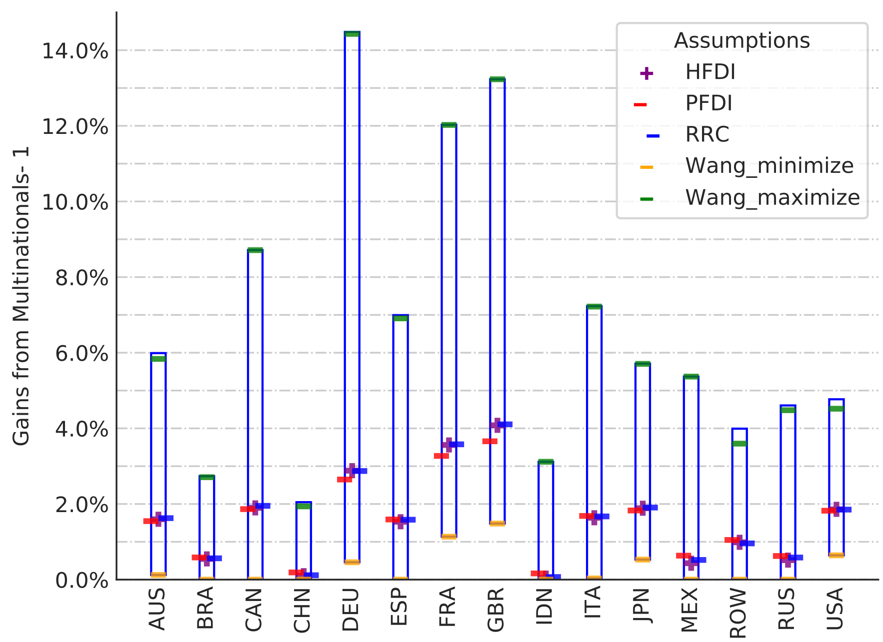
gains from trade in Germany range only from 3.1% to 6.1%. The low maximum of the interval is attributed to the fact that MP works as a substitute for trade and mitigates the loss from trade.

While the allocation of horizontal FDI achieves the maximum for gains from openness, the same is not the case for the gains from trade as the allocation of horizontal FDI attains a low value in the interval. For example, gains from trade in Germany for the horizontal FDI allocation are 3.1%, which is at the lower end of the interval (the minimum value is 3.05%). In the allocation of horizontal FDI, trade and MP are two distinct and substitutable ways to serve foreign countries. Given that horizontal FDI exhibits substitution between trade and MP, the gains from trade should be small. Wang's model achieves a narrower range compared with the interval from the triangulation. In the UK, while the interval from the triangulated set ranges from 1.7% to 3.7%, the value in Wang's model only ranges from 1.7% to 3.1%. This result highlights the restrictiveness of Wang's model in terms of gains from trade.

5.3 Gains from multinationals

Regarding gains from multinationals, I show the interval and the values from five allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo and Rodríguez-Clare's model, (iii) horizontal FDI allocation, (iv) allocation from Wang's model that maximizes gains from the multinationals and (v) allocation from Wang's model that minimizes gains from multinationals. Figure 3 shows the result.

Figure 3: Gains from multinationals



The range of the interval for gains from multinationals is as wide as (or even wider than) that of gains from openness. For example, for Germany, gains from multinationals range from 0.4% to 14.4% (the gains from openness range from 4.5% to 14.5%) and in France, gains from multinationals range from 1.1% to 12.0% (the gains from openness range from 4.6% to 12.0%).

In some countries—notably Brazil, Canada, China, Indonesia, Italy, Mexico, Russia and Spain—the lower end of gains from multinationals is zero. The zero gains from multinationals happens when the consumer in the country does not consume goods produced by affiliates.[19] In this case,

[19]This is also due to fixed wage. Without the changes in wages, only the price change in the consumption basket matters.

there is no loss of consumption due to this counterfactual experiment.

In contrast, in most countries, the maximum of gains from multinationals is equal to the maximum gains from openness. If the country only imports the good produced by affiliates, abandoning MP also abandons trade.[20] Thus, the range of the interval for gains from multinationals is wide as that of gains from openness and is significantly wider than that of gains from trade.

Similar to gains from trade, gains from multinationals from the horizontal FDI allocation are in the lower end of the interval. In the UK, the gains from multinationals for the allocation of horizontal FDI are 4.0%, where the interval ranges from 1.4% to 13.0%. The explanation is similar to that of gains from trade. In horizontal FDI allocation, trade and MP are two distinct ways to serve the market, hence trade mitigates the loss from abandoning multinationals. Both gains from multinationals of the proportional export platform FDI allocation and that of Ramondo and Rodríguez-Clare's model are at the lower side of the interval. Similar to gains from trade, Wang's model has a smaller range of gains from multinationals. One example is Germany, in which the interval ranges from 0.4% to 14.4% while Wang's model's value ranges from 0.4% to 13.2%.

Overall, gains from openness, gains from trade, and gains from multinationals exhibit economically sizable indeterminacy (range of the interval). Numerical results indicate that the results from the previous studies, which impose various assumptions on the allocation, must be treated with caution.

5.4 Including intermediate goods in the model and the triangulation

We can extend the triangulation, the model, and the gains from openness to incorporate intermediate goods. The extended model generalizes the model developed by Li, 2021 to incorporate more flexible patterns to input-output linkage across MP. Appendix D shows a detailed explanation of the triangulation, the model, and gains from openness. Incorporating intermediate goods shows the interval of gains from openness is significantly wider than the interval without intermediate goods.

[20]The upper end is not the consequence of partial equilibrium. When trade is abandoned, a general equilibrium effect does not exist.

6 Policy Evaluation

In this section, I show how this framework can be used for policy evaluations. Before discussing the plicy of interest, I first formulate an exact hat-algebra to compute counterfactual equilibrium in terms of proportional change. I denote $\hat{x}$ a proportional change in a variable x. Formally, $\hat{x} = x'/x$ where x is a value in the observed equilibrium, and x' is the corresponding value in the counterfactual equilibrium. A particular policy experiment is $\hat{\boldsymbol{\tau}}$, a proportional change in $\boldsymbol{\tau}$. For example, reducing the cost of trade from country o to country q by $(1-z)\%$ is formulated as:

$$\hat{\tau}_{ilm} = \begin{cases} z \text{ if } l = o,\ m = q \\ 1 \text{ otherwise .} \end{cases}$$

Given $\hat{\boldsymbol{\tau}}$, exact hat-algebra solves a system of nonlinear equations:

$$\begin{aligned}
\hat{p}_{ilm} &= \hat{w}_l \hat{\tau}_{ilm} \\
\hat{\pi}_{ilm} &= \frac{\hat{P}_{im}^{-\theta} \hat{p}_{ilm}^{-\theta/(1-\rho)}}{\sum_{j=1}^{N} \pi_{jm} \hat{P}_{jm}^{-\theta} \sum_{k=1}^{N} \pi_{km|i} \hat{p}_{ikm}^{-\theta/(1-\rho)}} \\
\hat{P}_{im} &= \left(\sum_{k=1}^{N} \pi_{km|i} \hat{p}_{ikm}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta} \\
\hat{P}_{m} &= \left(\sum_{i=1}^{N} \pi_{im} \hat{P}_{im}^{-\theta} \right)^{-1/\theta} \\
X'_m &= \hat{w}_m Y_m + D_m \\
\hat{w}_m Y_m &= \sum_{j=1}^{N} \sum_{k=1}^{N} \hat{\pi}_{kmj} \pi_{kmj} X'_j
\end{aligned}$$

where:

$$\begin{aligned}
\pi_{ilm} &= \frac{X_{ilm}}{X_m} \\
\pi_{im} &= \frac{\sum_{k=1}^{N} X_{ikm}}{X_m} \\
\pi_{im|l} &= \frac{X_{ilm}}{\sum_{k=1}^{N} X_{ikm}}.
\end{aligned}$$

I denote the variables in the counterfactual equilibrium $\hat{\boldsymbol{x}}$ and the system of nonlinear equations $H(X, \hat{\boldsymbol{\tau}}, \hat{\boldsymbol{x}}) = 0$. I focus on the interval of changes in the real wage ($\hat{W}_q = \hat{w}_q/\hat{P}_q$), where I denote $\hat{\mathbb{W}}_q \equiv [\hat{W}_q^L(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}}), \hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}})]$. The maximum $\hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}})$ can be calculated by solving a following problem:

$$
\begin{aligned}
\hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}}) = maximize_{\boldsymbol{X}, \hat{\boldsymbol{x}}}\ & \hat{w}_q/\hat{P}_q \\
s.t.\ M_{il} = & \sum_{m=1}^{N} X_{ilm}\ \forall\ i, l \\
T_{lm} = & \sum_{i=1}^{N} X_{ilm}\ \forall\ l, m \\
X_{ilm} \geq & \ 0\ \forall\ i, l, m. \\
H(X, \hat{\tau}, & \hat{x}) = 0,
\end{aligned}
$$

and the minimum can be calculated in a similar way. For each country, I solve these problems to obtain the interval.

6.1 Policy experiment

With the exact hat-algebra in hand, I evaluate the policy. I focus on what the President of the United States, Joe Biden, proposed in his 2020 U.S. presidential campaign. On September 9th, 2020, Joe Biden proposed a 10% surtax on goods produced by U.S. affiliates abroad that are sold to the U.S. consumer (Wilkie, 2020). This policy aims to discourage offshoring by U.S. multinationals and bring jobs back to the U.S. This policy is an excellent example to discuss in this model because this policy targets the specific intersection of trade and MP. I evaluate the policy in three steps: First, I show the interval of real wage change owing to this policy. In the second step, I incorporate the BEA (U.S. Bureau of Economic Analysis) data to show how the additional data shrink the interval. In the third step, I propose an alternative policy and compare the welfare effect with the original policy.

In my model, the policy is modeled as 10% increase in the cost of production when firms from the U.S., produced goods in foreign countries and is sold to the U.S. consumer:

$$\hat{\tau}_{ilm} = \begin{cases} 1.1 & \text{if } i = U.S., l \neq U.S., m = U.S. \\ 1 & \text{otherwise.} \end{cases}$$

For the U.S. consumer, the effect of this policy is ambiguous. On the one hand, the policy increases the price of offshored goods, which will negatively affect the U.S. consumer. On the other hand, by suppressing the demand for foreign labor and bringing the production back to the U.S., this policy can improve terms of trade for the U.S. The net effect of the policy should be calculated by solving the counterfactual equilibrium. A similar argument can be made for the consumer in other countries.

The welfare prediction is shown in terms of intervals. For each country, I calculate the interval of welfare change (change in the real wage). I also calculate the welfare change for five allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo and Rodríguez-Clare's model, (iii) horizontal FDI allocation, (iv) allocation from Wang's model that maximizes gains from the multinationals and (v) allocation from Wang's model that minimizes gains from multinationals.

Figure 4: Biden's policy : Benchmark

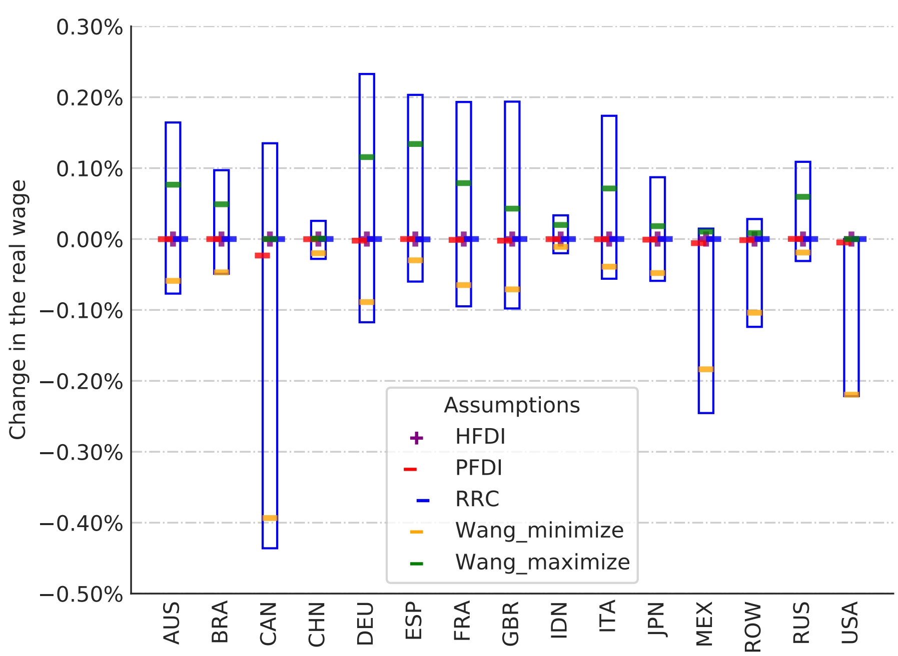


Figure 4 shows the result. There are four noticeable features in the result. First, the policy has no effect in the case of horizontal FDI allocation, because there is no offshoring on the horizontal FDI allocation. Since the proportional export platform FDI and the Ramondo and Rodríguez-Clare's allocation are similar to the allocation of horizontal FDI, these three have similar predictions. Second, Wang's assumption significantly narrows the range. For example, the maximum welfare gain for Japan is 0.02% when Wang's assumption is imposed, while it is 0.09% with only the triangulation. His assumption even provides a qualitative implication to the policy. For example, for Canada, Wang's assumption implies that Canada never gains from this policy, while Canada may gain more than 0.1% if the assumption is violated. Third, for countries other than the U.S., the effect of the policy could either be positive or negative. This suggests that the data on bilateral trade and MP are insufficient to judge if the policy benefits other countries. Fourth, the prediction on the U.S. is clear in this model: The interval ranges from 0% to -0.2%, which implies that the policy does not benefit the U.S. consumer. The direct effect on the consumer price outweighs the terms of trade improvement.

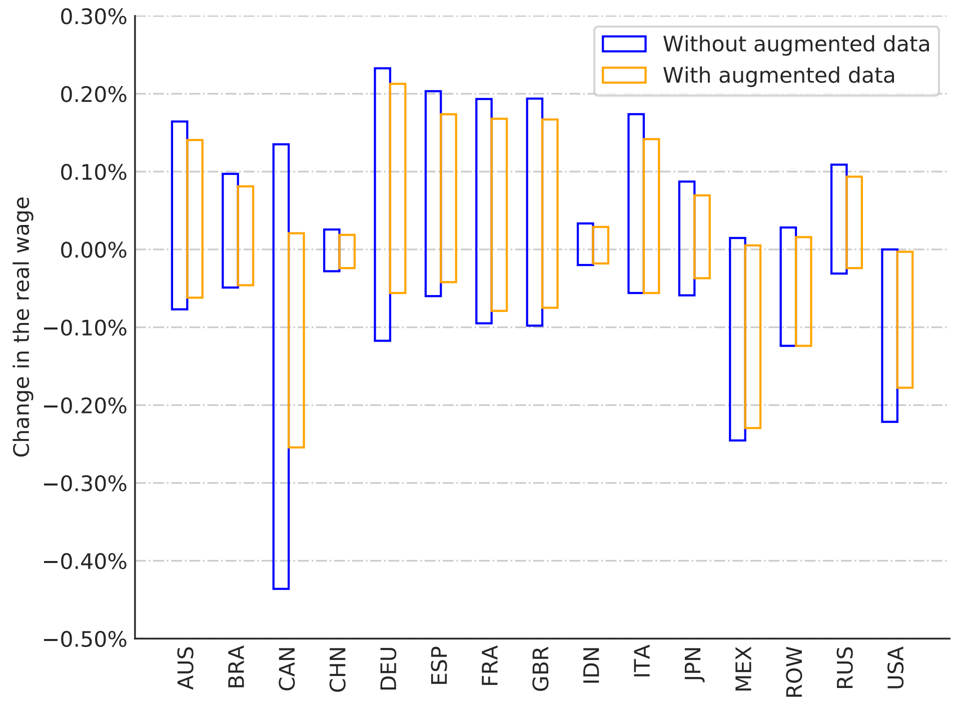
To anatomize the variation in the outcome, I examine two allocations: the allocation that maximizes the welfare gain of the U.S. and the allocation that minimizes the welfare gain of the U.S. consumer. These two allocations take two extremes in terms of U.S. multinationals' offshoring. In the allocation for the maximum gain (no change in the U.S. real wage), there is no offshoring in the allocation. For the minimum gain (0.2% loss in the U.S. real wage), 82% of U.S. imports are offshored goods. We can see from this number that the amount of offshoring by U.S. multinationals is crucial for ascertaining the consequence of the policy.

6.2 Narrowing the interval using additional data

To further narrow the interval, I add information on U.S. multinationals' offshoring activity. BEA provides the data on the sales of U.S. affiliates abroad to the U.S. Specifically, I collect the data for the affiliates in Canada, China, Germany and Japan ($\{X_{USA,l,USA}\}_{l=CAN,CHN,DEU,JPN}$) and include them as additional constraints on the allocation. Figure 5 shows the result. I show two intervals with and without additional data on the allocation.

There are three points worth discussing. First, the intervals with additional data on the allocation have a narrower range. The additional data are most informative for Canada; without additional data, the change in welfare ranges from -0.44% to 0.13%, while with the additional data, it ranges from -0.26% to 0.019%. The reduction is most significant in Canada because there are significant volumes of trade and MP between the two countries, and I directly added the data on Canada to restrict the degree of offshoring done by the U.S. firms. Second, the reduction of the range occurs in the countries without additional data as well as in other countries. The general equilibrium feature of the model propagates the information to narrow the policy effect for countries without additional data. Third, the additional data further shows that the U.S. consumer loses from this policy in terms of welfare; at best, the U.S. consumer loses approximately 0.005%, and at worst, they lose approximately 0.2%.

Figure 5: Biden's policy : Augmented data



6.3 Is there a better policy?

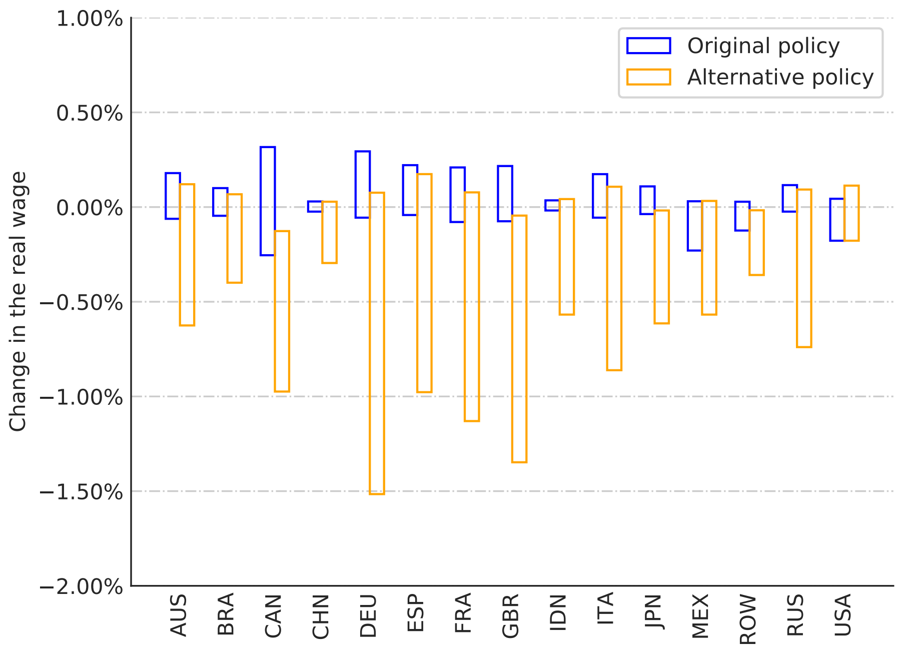
I further explore whether there is a possible policy that is more beneficial to the U.S. consumer. A simple alternative to this policy is to penalize U.S. multinationals' production, regardless of the sales destination. The alternative policy can be described as follows:

$$\hat{\tau}_{ilm} = \begin{cases} 1.1 & \text{if } i = U.S., l \neq U.S. \\ 1 & \text{otherwise.} \end{cases}$$

I use the augmented data and derive the welfare intervals for each country. Figure 6 compares the intervals of the original policy and the alternative policy. Most of the countries, other than the U.S., are likely to be worse off when the alternative policy is implemented.[21] The welfare change from the alternative policy tends to have a lower maximum and a lower minimum than the original policy. The interval is also wider for the alternative policy. This wider interval is because alternative policy is agnostic on who bears the direct cost of the policy, while the direct cost of the original policy is concentrated on the U.S. consumer. The policy is likely to be better for the U.S. Both policies produce identical welfare loss for the worst case, while the alternative policy has a brighter possibility; at best, the U.S. may gain approximately 0.1% from this policy. The interval suggests that the alternative policy may be better for the U.S. while potentially harmful for other countries.

[21] Note that this is not an allocation-wise comparison. There may be an allocation that the original policy is more harmful than the alternative policy.

Figure 6: Bident's policy : Comparison



7 Conclusion

In this paper, I develop a quantitative framework of trade and multinational production (MP) for counterfactual predictions and policy assessments. I develop an empirical approach — *triangulation* and a quantitative model — that allows MP to occur for various motives and synthesize models in the literature as special cases. Triangulation and the model provide an interval of counterfactual outcomes, including outcomes from past studies. The interval shows possible outcomes without imposing any assumption on the interaction between trade and MP. The interval summarizes the indeterminacy of the counterfactual outcomes due to the lack of complete data.

I analyze various models and counterfactual experiments using triangulation and the quantitative model. First, I use the interval to theoretically analyze the quantitative role of assumptions in the literature. I show that, for any country, gains from openness (from both trade and MP) are at a maximum with pure horizontal FDI and a minimum with pure vertical FDI. Second, I use the actual data to quantify various counterfactual experiments. Empirical results show that a wide range of outcomes is consistent with the data; for example, the welfare gains from openness in

Germany range between 4.5% and 14.5%. While the incompleteness of the data leaves open a wide range of outcomes, the available information can still bring clear judgment on policy evaluation. In this paper, I show that the policy penalizing U.S. multinationals for offshoring does not benefit the U.S.

One limitation of triangulation is its lack of scalability. Except for the case where we know the allocation that characterizes the interval (e.g., Gains from Openness), calculating the bound is computationally expensive, especially for the general policy counterfactual experiments using exact hat-algebra. Searching for powerful computational methods or developing a convenient approximation of exact hat-algebra would greatly improve the usability of triangulation.

Even with the computational burden, triangulation is a valuable tool to calibrate the model and perform counterfactual analysis. The concept is helpful beyond this context. There are many economic issues where only aggregate data are available, while disaggregated data is crucial to measure economic outcomes. Triangulation will be an excellent approach for such situations.

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Appendix A Quantitative model

Appendix A.1 Constructing a model with arbitrary allocation

Here I show that given $\theta, \rho, \boldsymbol{L}$ and $\boldsymbol{D}$, for any $\boldsymbol{X}$, there exists $\boldsymbol{\tau}$ such that $\boldsymbol{X}$ is an equilibrium outcome. First, I set $\boldsymbol{w}$ as

$$w_l L_l = \sum_m \sum_k X_{klm} + D_l,$$

so that the economy satisfies goods market equilibrium. Further, for the elements of $\boldsymbol{X}$ where the value is zero, I set $\tau_{ilm} = \infty$. $p_{ilm} = \infty$, eliminating the expenditure for such goods. For the non-zero elements of $\boldsymbol{X}$, I set τ_{ilm} so that the demand of goods is what is observed in $\boldsymbol{X}$. Specifically, I set $\tau_{ilm} = p_{ilm}/w_l$ so that

$$X_{ilm} = \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}} X_m.$$

For each market m, there are N^2 equations (ignoring the zeros) and N^2 parameters τ_{ilm}. The $\boldsymbol{\tau}$ satisfying the equation can be found through following the steps in the appendix of Berry (1994).

Appendix A.2 The equivalence between proportionality assumption and the separability assumption

With parameter $\rho = 0$, the allocation $\boldsymbol{X}^{PFDI}$ is equivalent to $\boldsymbol{X}^{RRC}$. Formally stated, this implies

Proposition Appendix A.1. *If $\rho = 0$ and $\boldsymbol{X}^{RRC} \in \mathbb{X}$, $\boldsymbol{X}^{RRC} = \boldsymbol{X}^{PFDI}$.*

Proof. Note that Ramondo and Rodríguez-Clare (2013) assumes $\tau_{ilm} = \gamma_{il}\zeta_{lm}$. Combined with

$\rho = 0$, this implies

$$X_{jlm} = \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{ilm}$$

Further, I show that this implies

$$X_{ilm} = \frac{M_{il}}{\sum_{j}^{N} M_{jl}} T_{lm}$$

This is satisfied by the following reformulation:

$$\begin{aligned}
\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}} T_{lm} &= \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} X_{jln}} \sum_{j=1}^{N} X_{jlm} \\
&= \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{iln}} \left(\sum_{j=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{ilm}\right) \\
&= X_{ilm} \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{iln}} \sum_{j=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \\
&= X_{ilm} \frac{\sum_{j=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \sum_{n=1}^{N} X_{iln}} \\
&= X_{ilm}
\end{aligned}$$

which implies for any i, l, m, $X_{ilm}^{PFDI} = X_{ilm}^{RRC}$. □

Appendix A.3 Wang (2021) and canonical theories

Here I show a simple example that $\boldsymbol{X}^{HFDI}$ and $\boldsymbol{X}^{VFDI}$ are not in $\mathbb{X}^{Wang}$ while they are in the triangulated set. Consider three symmetric countries a, country b and country c. The flow of MP and trade are $M_{il} = M_f > 0, T_{lm} = T_f > 0$ for all $i \neq l$ and $l \neq m$. The domestic production and trade flow are same across countries, and are noted as $M_{ii} = M_d$, $T_{ii} = T_d$ $\forall i$. I assume:

$$M_d > 2T_f$$

$$T_f > M_f$$

which implies that both horizontal FDI and vertical FDI are in the triangulated set. Furthermore, this implies that there must be a positive export from headquarters to other countries for both allocations. Both horizontal FDI assumption and vertical FDI assumption implies $X_{abc} = 0$. Then the separability assumption implies:

$$\gamma_{ab}\zeta_{bc}\xi_{ac} = \infty.$$

This means at least one of the separable components must be arbitrarily large. Think of the case with $\zeta_{ab} = \infty$. This indicates $M_{ab} = M_f = 0$, which contradicts the observed MP. Similarly, $\zeta_{bc} = \infty$ indicates $T_{bc} = T_f = 0$, which contradicts the observed trade flow. The only possibility left is $\xi_{ac} = \infty$. However this requires $X_{aac} = 0$. Given $M_d > 2T_f$, the horizontal FDI allocation implies $X_{aac} > 0$ which contradicts $\xi_{ac} = \infty$. Similarly, given $T_f > M_f$, the vertical FDI assumption implies $X_{aac} > 0$, which contradicts $\xi_{ac} = \infty$. Therefore, both horizontal FDI allocation and vertical FDI allocation are not in $\mathbb{X}^{Wang}$ while they are in $\mathbb{X}$.

Appendix B Characeterizing counterfactuals

Appendix B.1 Varying parameters

I discuss the consequence of varying the parameter θ and ρ. First, for any allocation, $GO(\boldsymbol{X})$, $GT(\boldsymbol{X})$ and $GM(\boldsymbol{X})$ are non-increasing in θ and ρ. Because they are both elasticities of substitution between goods, the higher the value is, the lower the gains are. The maximum and the minimum of the intervals of these three gains will be lower when θ or ρ is higher. Changing θ does not change the allocations that maximize or minimize the gains; hence the intervals for different θ can be easily calculated. Changing parameter ρ may change the allocations.

There is a better insight for the intervals for gains from openness. I first discuss the role of ρ on the intervals. Note that the allocations $\boldsymbol{X}^{HFDI}$, $\boldsymbol{X}^{VFDI}$ and $\boldsymbol{X}^{MVFDI,q}$ do not depend on the parameters θ and ρ. In horizontal FDI allocation, gains from openness do not depend on ρ:

$$GO_q\left(\boldsymbol{X}^{HFDI}(\boldsymbol{T}, \boldsymbol{M})\right) = \left(\frac{X_{qqq}^{HFDI}}{X_q}\right)^{-1/\theta}.$$

In horizontal FDI allocations, there is no offshoring. Countries do not substitute offshored goods (X_{qlq}) for domestic goods (X_{qqq}); hence ρ does not affect gains from openness in $\boldsymbol{X}^{HFDI}(\boldsymbol{T}, \boldsymbol{M})$. This is not true for $\boldsymbol{X}^{VFDI}$ and $\boldsymbol{X}^{MVFDI,q}$. Because there are positive amounts of offshoring in these allocations, gains from openness calculated from these allocations are decreasing in ρ. Therefore, higher ρ implies the same maximum but the lower minimum of gains from openness.

Unfortunately for exact hat-algebra, there is no clear prediction on the consequences of varying parameters. For different parameters, the allocation that maximizes (minimizes) the real wage must be calculated.

Appendix B.2 Characterizing the interval

Here I consider the condition of the interval to be bounded. I redefine $\mathbb{F}$ to allow F to have smaller domain than $\boldsymbol{X}$:

$$\mathbb{F}(\boldsymbol{T}, \boldsymbol{M}) \equiv \{V | V = F(\boldsymbol{X}) \mid \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F)\}.$$

where $Dom(F)$ is a domain of the function F. The most general version of the statement is:

Proposition Appendix B.1. *If F is continuous on $\boldsymbol{X}$ and $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F)$ is connected, then $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is an interval (may have finite endpoints).*

Proof. Because $Dom(F) \cap \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is connected and F is a continuous function, $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ must be connected (this is a multivariate extension of intermediate value theorem). This implies that $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is an interval. □

If $Dom(F)$ is larger than $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$, then, I can further show that $\mathbb{F}$ is an closed interval.

Corollary Appendix B.1.1. *If F is a continuous on $\boldsymbol{X}(\boldsymbol{T}, \boldsymbol{M})$ and $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \subset Dom(F)$, then $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is a closed interval.*

Proof. Note that $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F) = \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$. The triangulated set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is a set defined by a system of linear equations (without strict inequality). This indicates that the set is a connected closed convex set. Because F is continuous, by the Weierstrass Extreme Value Theorem, $\mathbb{F}$ has both a maximum F^U and a minimum F^L. Since the set is connected and F is a continuous function, $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ must be connected. This implies that $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is a closed interval $[F^L, F^U]$. □

I move on to the specific cases:

Corollary Appendix B.1.2. $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M}) =\equiv \{GO_q(\boldsymbol{X}) \mid \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)\}$ *is an interval.*

Proof. GO_q is not defined in some allocations. Specifically, GO_q takes finite value (only) when $X_{qqq} > 0$ (If $X_{qqq} = 0$ then $GO_q = \infty$). Then $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is defined by these equations:

$$
\begin{aligned}
M_{il} &= \sum_{m=1}^{N} X_{ilm} \ \forall\ i, l \\
T_{lm} &= \sum_{i=1}^{N} X_{ilm} \ \forall\ l, m \\
X_{ilm} &\geq 0 \ \forall\ i, l, m \\
X_{qqq} &> 0.
\end{aligned}
$$

The set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is a set defined by a system of linear equations (which includes both inequalities and strict inequalities). The set is a convex set which is connected. By proposition B.2.1, $\mathbb{GO}(\boldsymbol{T}, \boldsymbol{M})$ is an interval. □

In addition, it is straightforward to verify whether $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ is bounded or not.

Corollary Appendix B.1.3. *If there is an allocation* $\boldsymbol{X}$ *such that* $X_{qqq} = 0$*, then* $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ *is unbounded from above. If thre is no such allocation in the triangulated set, then* $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ *is bounded.*

Proof. Recall that gains from openness are decreasing function of X_{qqq} and can be arbitrary large by choosing a small value of X_{qqq}. As long as $X_{qqq} > 0$, gains from openness take a finite value (The second part of the corollary is verified). Suppose there exists an allocation such that $X_{qqq} = 0$. Think of another allocation $\boldsymbol{X'}$ in $(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$. Because $(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is convex, any convex combination of $\boldsymbol{X}$ and $\boldsymbol{X'}$ is in the set. By choosing the allocation close enough to $\boldsymbol{X}$ (by changing the convex weight), one can attain arbitrary small X_{qqq}. and arbitrary large value of $GO_q(\boldsymbol{X})$. This implies that $\mathbb{GO}_q$ is unbounded from above. □

For the exact hat-algebra, $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \subset Dom(F)$. Therefore, the counterfactual outcomes consistent with the data constitutes a closed interval.

Appendix C Computation

I calculate intervals from gains from openness by deriving gains from openness in the horizontal FDI allocation and the modified vertical FDI allocation. I calculate gains from trade and gains from multinationals by numerically solving a constrained optimization problem. For this purpose, I use the NEOS server, which is an internet-based client-sever application that provides various solvers for optimization problems. I use two solvers provided in the NEOS server.

The first solver is Knitro, which combines interior-point methods and active-set methods for nonlinear programming. An advantage of Knitro is that it provides a solution within the time allotted (an eight hours limit for the NEOS server). While useful, this does not guarantee that the solution will be (globally) optimal. The second solver I use is BARON, which uses various techniques of the branch-and-reduce method to solve global optimization problems. The advantage of this solver is that it provides the upper bound and the lower bound for each problem. The disadvantage of this solver is that it may require a massive amount of time and memory. In some problems BARON does not provide a solution as there is not enough memory for BARON to solve the problem.

Given the constraint on the computational resource, I supplement the solution of BARON with the solution of Knitro. Specifically, if BARON solves the problem, I use the value provided by BARON. If BARON cannot solve the problem within the limit of the computational resource, I use the solution provided by Knitro. The results with the upper bound and the lower bound are shown in figure 7 and figure 8 . For gains from trade there are some cases that the bound is wider than the interval. However, the discrepancy is not too large; at maximum discprepancy between the interval and the bound is 2.0%, which is for the uppder bound for Germany. For gains from multinationals, the interval and the bounds conincide.

For exact hat-algebra, I only use the solution by Knitro. The problem seems to be too complicated for BARON; most of the time BARON does not provide a solution within the limitation of time and memory.

Figure 7: Gains from trade: Bounds

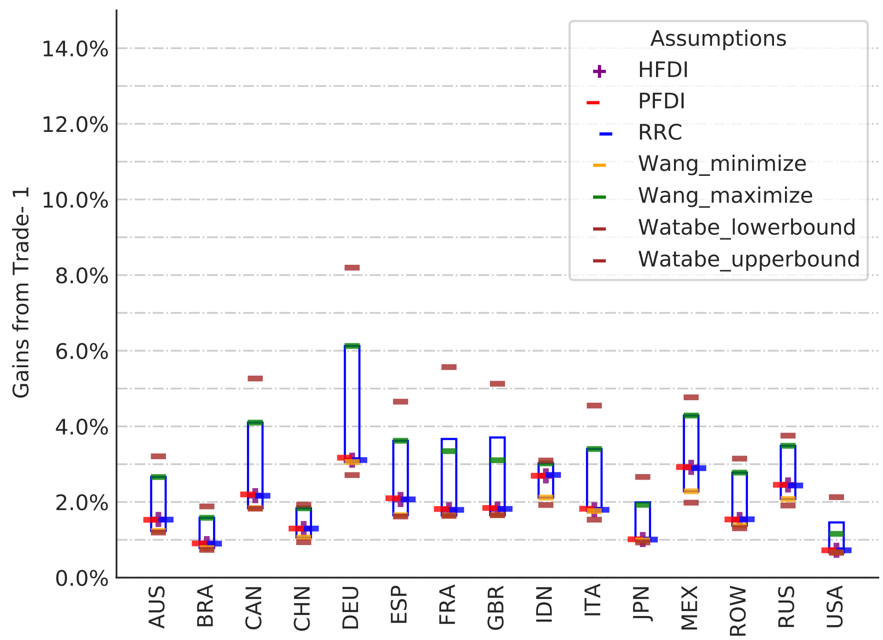
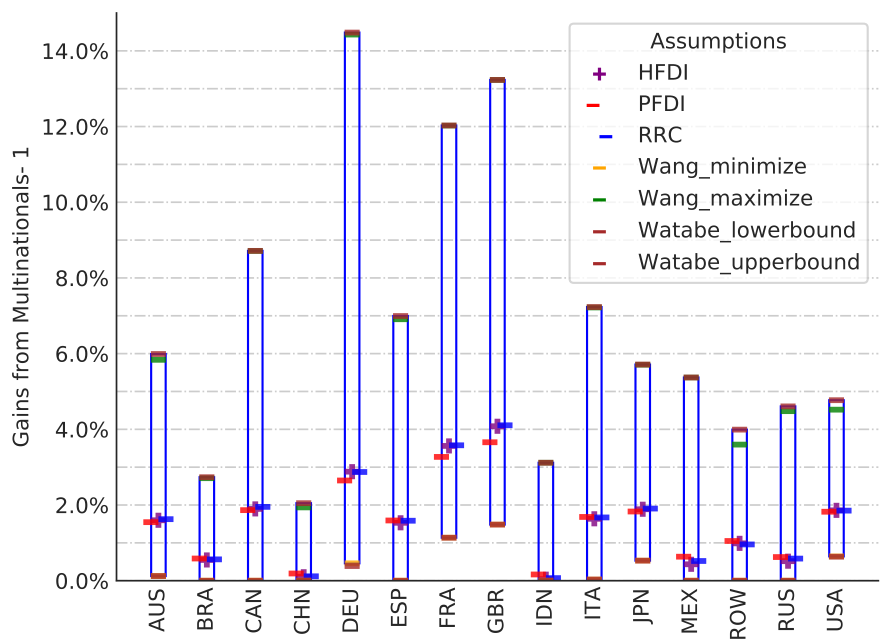


Figure 8: Gains from multinationals: Bounds



Appendix D Incorporating intermediate goods

I discuss the triangulation with intermediate goods and the model associated with it. In addition to the origin country i of the exporting firm, for intermediate goods, there is country n, the origin of the importing firm. Denote the value of goods used as intermediate goods produced by firms from country i, produced in country l, delivered to country m and used by the firms from country n as $X_{ilnm,int}$. Similarly, denote the value of goods used as final consumption produced by firms from country i, produced in country l and delivered to country m as $X_{ilm,f}$.

Appendix D.1 Triangulation

I use global input-output tables and MP data for the triangulation. Accounting identities for MP are:

$$M_{il} = \sum_{n=1}^{N} \sum_{m=1}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f}$$

where M_{il} is a summation of the production of intermediate goods and the production of final goods. The global input-output table records $T_{lm,int}$, a flow of intermediate goods from country l to country m; $T_{lm,f}$ a flow of final goods from country l to country m. The accounting identities for the global input-output tables are:

$$\begin{aligned} T_{lm,int} &= \sum_{i=1}^{N} \sum_{n=1}^{N} X_{ilnm,int} \\ T_{lm,f} &= \sum_{i=1}^{N} X_{ilm,f}, \end{aligned}$$

where the trade flow of intermediate goods is a summation of $X_{ilnm,int}$ over the origin i of the exporting firm and the origin n of the importing firm. Similarly, the trade flow of final goods is a summation of $X_{ilm,f}$ of over the origin of the exporting firm i. $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$ denote the vector notations of the intermediate goods flow and the final goods flow, respectively. In addition to the non-negativity constraint for $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$, I restrict the value added to be non-negative:

$$\sum_{n}^{N}\sum_{m}^{N} X_{nmil,int} \leq \sum_{n}^{N}\sum_{m}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f}$$

The triangulated set $\mathbb{X}$ is a set of $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$ that satisfies the conditions above.

Appendix D.2 Special case

I propose a special case that identifies the allocation from the data. The allocation of proportional export platform with intermediate goods is expressed as follows:

$$\begin{aligned} X^{PFDI}_{ilnm,int} &= T_{lm,int}\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}}\frac{M_{nm}}{\sum_{j=1}^{N} M_{jm}} \\ X^{PFDI}_{ilm,f} &= T_{lm,f}\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}}. \end{aligned}$$

The first equation implies that the composition of intermediate goods is common across the importing firms regardless of their origin if they are located in the same production location. This allocation is always in the triangulated set. The accounting identities on MP are satisfied as:

$$\begin{aligned} \sum_{n=1}^{N}\sum_{m=1}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f} &= \sum_{m=1}^{N} \left(T_{lm,int} + T_{lm,f}\right)\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}} \\ &= M_{il}. \end{aligned}$$

The accounting identities on the global input-output tables are trivially satisfied. The non-negativity of the allocation and the value-added can be verified through a simple calculation.

Appendix D.3 Model

I develop a model that incorporates multinationals into the Armington model (Armington, 1969) with input-output linkage. This model generalizes the model developed by Li (2021)[22]. There are N countries in the economy with representative firms and consumers. A consumer earns wages from her labor and purchase goods. A consumer in country l provides L_i amount of labor inelastically.

[22]Li, 2021 assumes separability on $\boldsymbol{\tau}$ similar to Wang (2021)

As in the original model, the utility function of a representative consumer in country m is:

$$U_m = \left(\sum_{i=1}^{N} \left(\sum_{l=1}^{N} C_{ilm,f}^{\frac{\epsilon}{\epsilon+1}} \right)^{\frac{\epsilon+1}{\epsilon} \frac{\theta}{\theta+1}} \right)^{\frac{\theta+1}{\theta}} .$$

Denote I_m as a total income for the consumer in country m. The expenditure of the final goods $X_{ilm,f}$ is

$$X_{ilm,f} = \frac{P_{im,f}^{-\theta}}{\sum_{j=1}^{N} P_{jm,f}{}^{-\theta}} \frac{p_{ilm,f}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{ikm,f}^{-\theta/(1-\rho)}} I_m$$

where $P_{im,f} \equiv \left(\sum_{k=1}^{N} (p_{ikm,f}^{-\theta/(1-\rho)}) \right)^{-(1-\rho)/\theta}$ is the price index in country m for final goods produced by the firms from country i. The price index of final goods in country m is

$$P_{m,f} = \left[\sum_{i=1}^{N} P_{im,f}^{-\theta} \right]^{-1/\theta} .$$

Firms combine labor and intermediate goods in a Cobb-Douglas manner to produce both the final goods and the intermediate goods. The cost share of the labor input is β_{il} (the cost share of the intermediate goods is $1 - \beta_{il}$). The labor share β_{il} depends on firm origin i and production location l. Intermediate goods from different firm origins and production locations are aggregated into composite intermediate goods in a nested CES manner (similar to the utility function for the representative consumer). Denote τ_{ilnm}^{int} as the quantity of (Cobb-Douglas composite of) labor and composite intermediate goods required by firms from country i, produced in country l, used in the production of firms from country n and delivered to country m. Denote τ_{ilm}^{f} as the quantity of (Cobb-Douglas composite of) goods required for final consumption produced by firms from country i, produced in country l, delivered to country m.

Perfect competition implies the price is set to the marginal cost of production. The price of intermediate goods $p_{ilmn,int}$ (which is the price corresponding to $X_{ilnm,int}$) is

$$p_{ilnm,int} = \tau_{ilnm}^{int} w_l^{\beta_{il}} P_{il,int}^{1-\beta_{il}} ,$$

where

$$P_{nm,int} = \left(\sum_{j=1}^{N} P_{jnm,int}^{-\theta} \right)^{-1/\theta}.$$

$$P_{inm,int} = \left(\sum_{k=1}^{N} p_{iknm,int}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta}$$

and the expenditure on such intermediate goods is

$$X_{ilnm,int} = \frac{P_{inm,int}^{-\theta}}{\sum_{j=1}^{N} P_{jnm,int}{}^{-\theta}} \frac{p_{ilnm,int}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{iknm,int}^{-\theta/(1-\rho)}} (1-\beta_{nm}) \left(\sum_{j=1}^{N} \sum_{k=1}^{N} X_{nmjk,int} + \sum_{k=1}^{N} X_{nmk,f} \right).$$

Similarly, the price of final goods $p_{ilm,f}$ delivered to country m produce in country l by firms from country i is

$$p_{ilmn,int} = \tau_{ilm}^{f} w_l^{\beta_{il}} P_{il,int}^{1-\beta_{il}}.$$

Total income of the representative consumer in country l is the labor income from the production and the transfer D_l:

$$I_l = w_l L_l + D_l = (1-\beta_{il}) \left(\sum_{i=1}^{N} \sum_{m=1}^{N} \sum_{n=1}^{N} X_{ilnm,int} + \sum_{i=1}^{N} \sum_{m=1}^{N} X_{ilm,f} \right) + D_l$$

Denote the vector of variables in bold. Given $\boldsymbol{D}$, $\boldsymbol{L}$, $\boldsymbol{\beta}$, $\boldsymbol{\tau}_{int}, \boldsymbol{\tau}_f$, an equilibrium is a wage vector $\boldsymbol{w}$, a price vector $\boldsymbol{p}_{int}, \boldsymbol{p}_f$, a final goods allocation $\boldsymbol{X}_{int}$ and a intermediate goods allocation $\boldsymbol{X}_f$ that satisfy the consumer optimization, the producer optimization and the market clearing condition.

Appendix D.4 Gains from Openness

Gains from openness with intermediate goods is written as follows:

$$GO_q(\boldsymbol{X}_{int}, \boldsymbol{X}_f) = \left(\frac{\sum_{k=1}^{N} X_{qkq,f}}{I_q} \right)^{-1/\theta} \left(\frac{X_{qqq,f}}{\sum_{k=1}^{N} X_{qkq,f}} \right)^{-(1-\rho)/\theta}$$
$$\left(\frac{\sum_{k=1}^{N} X_{qkqq,int}}{\sum_{i=1}^{N} \sum_{k=1}^{N} X_{ikqq,int}} \right)^{-(1-\beta_{qq})/\beta_{qq}\theta} \left(\frac{X_{qqqq,int}}{\sum_{k=1}^{N} X_{qkqq,int}} \right)^{-(1-\beta_{qq})(1-\rho)/\beta_{qq}\theta}$$

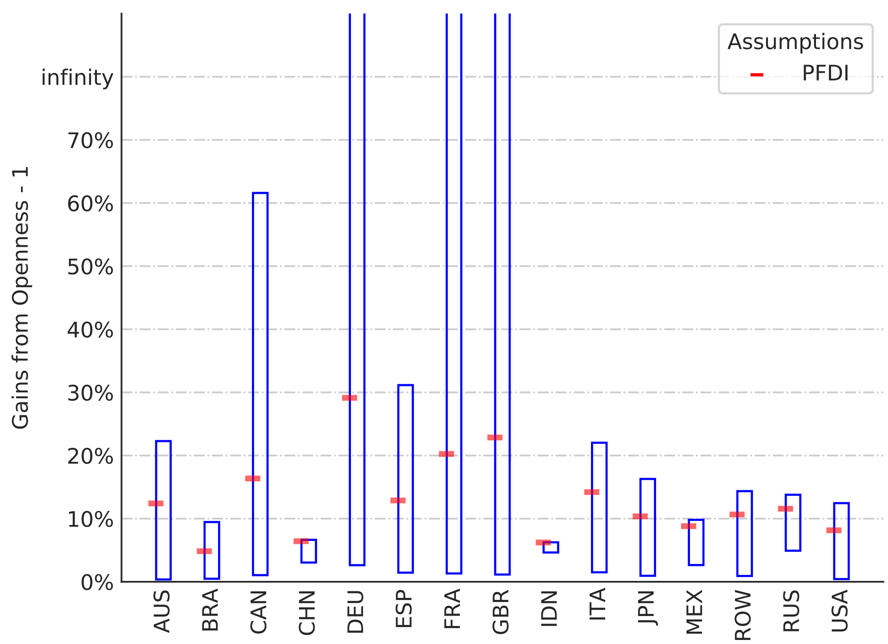
where β_{qq} is derived from the cost share:

$$\beta_{qq} = \frac{\sum_n^N \sum_m^N X_{nmqq,int}}{\sum_n^N \sum_m^N X_{qqnm,int} + \sum_{m=1}^N X_{qqm,f}}.$$

Appendix D.5 Quantification

I use the exact same data for the MP data. For the trade data, I disaggregate the bilateral trade flow into the intermediate goods flow and the final goods flow. I use the same parameter for θ and ρ. I calculate the intervals of gains from openness for 15 countries.[23] I include proportional export platform FDI as a special case and combine that with the interval. The result is shown in figure 9:

Figure 9: Gains from openness with intermediate goods



Note: Any interval ranging over the broken axis indicates that the interval is unbounded from above.

In countries like Germany, France and the U.K., the gains from openness are unbounded above. A simple example for the economy with infinite gains from openness is the case where the production of the domestic firm solely relies on intermediate inputs produced by foreign firms or being imported.

[23]While it is theoretically possible to perform policy experiments using exact hat-algebra for this model, obtaining an interval is too computationally costly (in terms of memory) to perform.

The autarky counterfactual makes it is impossible for domestic firms to use these intermediate goods; there will be no domestic goods to consume. Even for other countries with bounded intervals, the intervals tend to be wider. For example, with intermediate goods, the U.S. has gains from openness ranging from 0.04% to 12.4%, while without intermediate goods, the gains from openness for the U.S. ranges from 2.0% to 4.9%. The wider range suggests assuming particular allocation may be problematic. For many countries, gains from openness calculated from proportional export platform FDI is neither close to the maximum or the minimum, which implies the value from the assumption is not informative to predict the gains from openness.

Appendix E Derivations for gains from trade, gains from multinationals and gains from openness

In this section, I show how to derive gains from trade, gains from multinationals and gains from openness with intermediate goods. I denote x' a variable x in a counterfactual equilibrium. For gains from trade, this is for the counterfactual equilibrium without trade, and for gains from multinationals, this is for the counterfacual equilibrium without multinationals. I reiterate the additional notion used for the covenience:

$$\begin{aligned}\pi_{ilm} &= \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}} \\ \pi_{il} &= \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}} \\ \pi_{lm|i} &= \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}}.\end{aligned}$$

Appendix E.1 Gains from trade

If there is no trade, there is no exchange of labor embodied in trade; I can disregard the change in wages. I only need to track the changes in the price index. The price index of the country q is

$$P_q = \left[\sum_{i=1}^{N} P_{iq}^{-\theta}\right]^{-1/\theta}.$$

The counterfactual price index in trade autarky is

$$P'_q = \left[\sum_{i=1}^{N} P_{iq}'^{-\theta}\right]^{-1/\theta}.$$

$$P'_{iq} = p'_{iqq} = p_{iqq}.$$

Without trade, the only goods available from firms from country i are the goods produced in country q. Hence $P'_{iq} = p'_{iqq}$. Because the wage does not change, the price of such goods does not change. Notice that

$$\begin{aligned} P_{iq} &= \left(\sum_{l=1}^{N} p_{ilq}^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \\ &= p_{iqq}\left(\sum_{l=1}^{} \left(\frac{p_{ilq}}{p_{iqq}}\right)^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \end{aligned}$$

and because $\frac{X_{ilq}}{X_{iqq}} = \left(\frac{p_{ilq}}{p_{iqq}}\right)^{-\theta/(1-\rho)}$, I obtian

$$\begin{aligned} P_{iq} &= p_{iqq}\left(\sum_{l=1}^{N} \frac{X_{ilq}}{X_{iqq}}\right)^{-(1-\rho)/\theta} \\ &= p_{iqq}\pi_{qq|i}^{(1-\rho)/\theta}. \end{aligned}$$

I derive an expression for $\frac{p_{iqq}}{p_{qqq}}$:

$$\begin{aligned} \frac{\sum_{l=1}^{N} X_{qlq}}{\sum_{l=1}^{N} X_{ilq}} = \frac{\pi_{qq}}{\pi_{iq}} &= \left(\frac{P_{qq}}{P_{iq}}\right)^{-\theta} \\ &= \left(\frac{p_{qqq}}{p_{iqq}}\right)^{-\theta}\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)} \\ \left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta} &= \left(\frac{\pi_{iq}}{\pi_{qq}}\right)\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)}. \end{aligned}$$

Using these equations, the price index of country q is

$$\begin{aligned}
P_q &= \left[\sum_{i=1}^{N} p_{iqq}^{-\theta} \pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta} . \\
&= p_{qqq} \left[\sum_{i=1}^{N} \left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta} \pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \frac{\pi_{iq}}{\pi_{qq}} \left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)} \pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \frac{\pi_{im}}{\pi_{qq}} \pi_{qq|q}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq} \pi_{qq}^{1/\theta} \pi_{qq|q}^{(1-\rho)/\theta}
\end{aligned}$$

and the price index in trade autarky is

$$\begin{aligned}
P_q' &= \left[\sum_{i=1}^{N} p_{iqq}^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \frac{\pi_{iq}}{\pi_{qq}} \left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq} \pi_{qq}^{1/\theta} \pi_{qq|q}^{(1-\rho)/\theta} \left[\sum_{i=1}^{N} \pi_{iq} \pi_{qq|i}^{(1-\rho)}\right]^{-1/\theta} .
\end{aligned}$$

Then the gains from trade are

$$\begin{aligned}
GT_q = \frac{P_q'}{P_q} &= \left[\sum_{i=1}^{N} \pi_{iq} \pi_{qq|i}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N} \pi_{iq}^{\rho} \pi_{iqq}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilq}\right)^{\rho} X_{iqq}^{(1-\rho)}\right]^{-1/\theta} X_q^{1/\theta} .
\end{aligned}$$

Appendix E.2 Gains from multinationals

In the case of gains from multinationals, I cannot ignore the change in the wages because there is an exchange of labor through trade. I assume that there is no change in the wages, which can be justified by assuming a freely tradable numeraire sector, which fixes the wage. The price index of the country q is

$$P_q = \left[\sum_i P_{iq}^{-\theta} \right]^{-1/\theta}.$$

Now I state the counterfactual price index in multinational autarky. This is

$$P'_q = \left[\sum_{i=1}^{N} P'^{-\theta}_{iq} \right]^{-1/\theta}.$$

$$P'_{iq} = p_{iiq}.$$

Here $P'_{iq} = p'_{iiq}$ because without trade, the only goods available from a firm from country i is the goods produced in country i. The price of such goods does not change because the wage is fixed. Note that

$$\begin{aligned} P_{iq} &= \left(\sum_{l=1}^{N} p_{ilq}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta} \\ &= p_{iiq} \left(\sum_{l=1}^{N} \left(\frac{p_{ilq}}{p_{iiq}} \right)^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta}, \end{aligned}$$

and because $\frac{X_{ilq}}{X_{iiq}} = \left(\frac{p_{ilq}}{p_{iiq}} \right)^{-\theta/(1-\rho)}$, I obtain

$$\begin{aligned} P_{iq} &= p_{iiq} \left(\sum_{l=1}^{N} \frac{X_{ilq}}{X_{iiq}} \right)^{-(1-\rho)/\theta} \\ &= p_{iiq} \pi_{iq|i}^{(1-\rho)/\theta} \end{aligned}$$

I calculate $\frac{p_{iiq}}{p_{qqq}}$:

$$\begin{aligned}
\frac{\pi_{qq}}{\pi_{iq}} &= \left(\frac{P_{qq}}{P_{iq}}\right)^{-\theta} \\
&= \left(\frac{p_{qqq}}{p_{iiq}}\right)^{-\theta} \left(\frac{\pi_{qq|q}}{\pi_{iq|i}}\right)^{-(1-\rho)} \\
\left(\frac{p_{iiq}}{p_{qqq}}\right)^{-\theta} &= \left(\frac{\pi_{iq}}{\pi_{qq}}\right) \left(\frac{\pi_{qq|q}}{\pi_{iq|i}}\right)^{-(1-\rho)}.
\end{aligned}$$

The price index is already derived in the previous section, which is

$$P_q = p_{qqq}\pi_{qq}^{1/\theta}\pi_{qq|q}^{(1-\rho)/\theta}.$$

The price index for multinational autarky is

$$\begin{aligned}
P_q' &= \left[\sum_{i=1}^{N} p_{iiq}^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\left(\frac{p_{iiq}}{p_{qqq}}\right)^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\frac{\pi_{iq}}{\pi_{qq}}\left(\frac{\pi_{qq|q}}{\pi_{iq|i}}\right)^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq}\pi_{qq}^{1/\theta}\pi_{iq|q}^{(1-\rho)/\theta}\left[\sum_{i=1}^{N}\pi_{iq}\pi_{iq|i}^{(1-\rho)}\right]^{-1/\theta}.
\end{aligned}$$

Threfore, the gains from multinational are

$$\begin{aligned}
\frac{P_q'}{P_q} &= \left[\sum_{i=1}^{N}\pi_{iq}\pi_{iq|i}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N}\pi_{iq}^{\rho}\pi_{iiq}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N}\left(\sum_{l=1}^{N}X_{ilq}\right)^{\rho}X_{iiq}^{(1-\rho)}\right]^{-1/\theta} X_q^{-1/\theta}.
\end{aligned}$$

Appendix E.3 Gains from Openness with intermediate goods

I derive gains from openness with the model with intermediate goods. I denote x' a variable x in a counterfactual equilibrium. The price of intermediate goods $p_{qqqq,int}$ in autarky is:

$$p'_{qqqq,int} = w'_q(\tau'_{qqqq,int})^{1/\beta_{qq}}$$

so $P'_{qq,int} = w'_q(\tau'_{qqqq,int})^{1/\beta_{qq}}$. Now I look the price index in current observed equilibrium. The price index of the intermediate goods is

$$\begin{aligned}
P_{qqq,int} &= \left(\sum_{k=1}^{N} p_{qkqq,int}^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\left(\sum_{k=1}^{N} \frac{p_{qkqq,int}^{-\theta/(1-\rho)}}{p_{qqqq,int}^{-\theta/(1-\rho)}}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\left(\sum_{k=1}^{N} \frac{\pi_{qkqq,int}}{\pi_{qqqq,int}}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\pi_{qkq|q,int}^{(1-\rho)/\theta} \\
P_{qq,int} &= \left(\sum_{i=1}^{N} P_{iqq,int}^{-\theta}\right)^{-1/\theta} \\
&= P_{qqq,int}\left(\sum_{i=1}^{N}\left(\frac{P_{iqq,int}}{P_{qqq,int}}\right)^{-\theta}\right)^{-1/\theta} \\
&= P_{qqq,int}\left(\sum_{i=1}^{N} \frac{\pi_{iqq,int}}{\pi_{qqq,int}}\right)^{-1/\theta} \\
&= p_{qqq,int}\pi_{qqq|q,int}^{(1-\rho)/\theta}\pi_{qqq,int}^{1/\theta} \\
p_{qqqq,int} &= \tau_{qqqq,int}w_l^{\beta_{qq}}P_{qq,int}^{1-\beta_{qq}} \\
&= \tau_{qqqq,int}w_l^{\beta_{qq}}p_{qqq,int}^{1-\beta_{qq}}\pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta}\pi_{qqq,int}^{(1-\beta_{qq})/\theta}
\end{aligned}$$

Therefore,

$$\begin{aligned}
P_{qq,int} &= \tau_{qqqq,int}^{1/\beta_{qq}}w_l\pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta\beta_{qq}}\pi_{qqq,int}^{(1-\beta_{qq})/\theta\beta_{qq}}\pi_{qqq|q,int}^{-(1-\rho)/\theta}\pi_{qqq,int}^{1/\theta} \\
&= \tau_{qqqq,int}^{1/\beta_{qq}}w_l\pi_{qqq|q,int}^{(1-\rho)/\theta\beta_{qq}}\pi_{qqq,int}^{1/\theta\beta_{qq}}
\end{aligned}$$

and

$$\frac{P_{qq,int}}{P'_{qq,int}} = \pi_{qqq|q,int}^{(1-\rho)/\theta\beta_{qq}} \pi_{qqq,int}^{1/\theta\beta_{qq}}.$$

Using this equation to the final goods price index implies gains from openness is:

$$\begin{aligned}
GO_q = \frac{P_{q,f}}{P'_{q,f}} &= \frac{p_{qqq,f}\pi_{qq|q,f}^{(1-\rho)/\theta}\pi_{qq,f}^{\theta}}{p'_{qqq,f}} \\
&= \frac{\tau_{qqq,f} w_q^{\beta_{qq}} P_{qq,int}^{1-\beta_{qq}} \pi_{qq|q,f}^{(1-\rho)/\theta}\pi_{qq,f}^{\theta}}{\tau_{qqq,f} w_q^{\beta_{qq}} (P'_{qq,int})^{1-\beta_{qq}}} \\
&= \pi_{qq|q,f}^{(1-\rho)/\theta}\pi_{qq,f}^{\theta}\pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta\beta_{qq}}\pi_{qqq,int}^{(1-\beta_{qq})/\theta\beta_{qq}} \\
&= \left(\frac{\sum_{k=1}^N X_{qkq,f}}{I_q}\right)^{-1/\theta} \left(\frac{X_{qqq,f}}{\sum_{k=1}^N X_{qkq,f}}\right)^{-(1-\rho)/\theta} \\
&\left(\frac{\sum_{k=1}^N X_{qkqq,int}}{\sum_{i=1}^N \sum_{k=1}^N X_{ikqq,int}}\right)^{-(1-\beta_{qq})/\beta_{qq}\theta} \left(\frac{X_{qqqq,int}}{\sum_{k=1}^N X_{qkqq,int}}\right)^{-(1-\beta_{qq})(1-\rho)/\beta_{qq}\theta}
\end{aligned}$$

where β_{qq} is

$$\beta_{qq} = \frac{\sum_n^N \sum_m^N X_{nmqq,int}}{\sum_n^N \sum_m^N X_{qqnm,int} + \sum_{m=1}^N X_{qqm,f}}.$$